Compound Structures of Six New Chaotic Attractors in a Modified Jerk Model using *Sinh*$^{-1}$ Nonlinearity

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Abstract: Six new chaotic attractors in a modified single-coefficient jerk model are presented based on *Sinh*$^{-1}$ nonlinearity and six new values of the single coefficient. Compound structures of such chaotic attractors are revealed through the use of a control parameter *n* of a half-image operation. For an appropriate value of *n*, a positive *n* isolates a right half-image attractor, whereas a negative *n* isolates a left half-image attractor. Both images can be merged together as a compound structure.
Keywords: Chaos, Jerk moel, Compound structure, Sinh^{-1} nonlinearity.

1 Introduction

Studies of chaotic behavior in nonlinear systems and circuits have attracted great attention due to a variety of applications in science and technology. The best known electronic circuit exhibiting chaos is the Chua's circuit [1], [2], based on three first-order ordinary differential equations (ODEs). In contrast, Sprott [3] has alternatively proposed chaotic circuits based on a single third-order ODE in a "Jerk Model" with a single coefficient *K*, as shown in (1). The nonlinear component *G*(*x*) has been summarized in (2).

$$\frac{d^3x}{dt^3} + K\frac{d^2x}{dt^2} + \frac{dx}{dt} = G(x) \tag{1}$$

$$G(x) = \begin{cases} |x| - 2 & ; K = 0.6\ [3] \\ -6max(x,0) + 0.5 & ; K = 0.6\ [3] \\ -4.5sgn(x) + 1.2x & ; K = 0.6\ [3] \\ 2sgn(x) - 1.2x & ; K = 0.6\ [3] \\ 2tanh(x) - x & ; K = 0.19\ [5] \\ 3sin(x) - x & ; K = 1\ \ [6] \\ 6tan^{-1}(x) - 2x & ; K = 1\ \ [6] \\ 7tanh(x) - 2x & ; K = 1\ \ [6] \\ sgn(x) - 2x & ; K = 1\ \ [6] \end{cases} \tag{2}$$

The term "jerk" comes from the fact that in a mechanical system in which *x* is

Received: 2 June 2010 / Accepted: 12 January 2012

ISSN 2241-0503

the displacement, successive time derivatives of x are velocity, acceleration, and jerk [4]. Some of these jerk models have been implemented using current-feedback op-amps [7], [8]. In addition, other values of the single coefficient K have been presented using either Tan^{-1} nonlinearity [9] or Sin^{-1} nonlinearity [10]. Recently, compound structures of chaotic attractors based on the single-coefficient jerk model [9], [10] and others [11], [12], [13] have been reported.

In this paper, six new chaotic attractors in a modified single-coefficient jerk model are proposed based on $Sinh^{-1}$ nonlinearity and six new values of the single coefficient. In addition, compound structures of the six chaotic attractors are also demonstrated.

2 A Modified Single-Coefficient Jerk Model

Figure 1 shows an implementation of the jerk model described in (1) and (2) where the single coefficient K and the nonlinearity $G(x)$ can now be modified. By using new nonlinearity $Sinh^{-1}(x)$, six new values of K and $G(x)$ are proposed, as shown in (3).

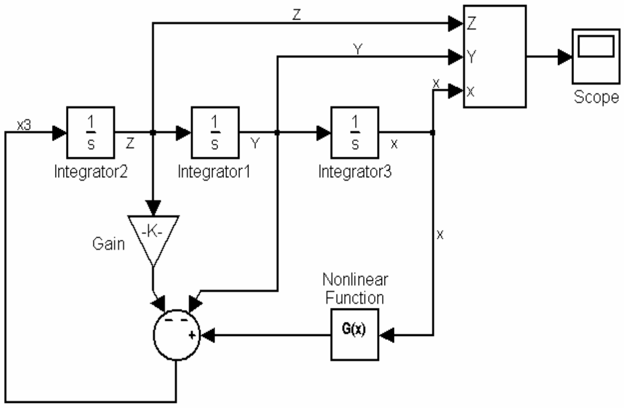

Fig. 1. A Single-Coefficient Jerk Model.

$$G(x)=\begin{cases} G_1(x)=+4Sinh^{-1}(x)-x; K=0.24 \\ G_2(x)=+5Sinh^{-1}(x)-x; K=0.26 \\ G_3(x)=+6Sinh^{-1}(x)-x; K=0.32 \\ G_4(x)=-4Sinh^{-1}(x)+x; K=0.19 \\ G_5(x)=-5Sinh^{-1}(x)+x; K=0.21 \\ G_6(x)=-6Sinh^{-1}(x)+x; K=0.23 \end{cases} \quad (3)$$

3 Compound Structures of New Chaotic Attractors

In the new systems shown in (1) and (3), compound structures [9]-[13] can be demonstrated using a half-image operation to obtain either a left- or a right-half-image attractor, each of which can be merged together as a compound structure. Such a half-image attractor can be revealed through the use of a control parameter *n* of the form:

$$\frac{d^3x}{dt^3} + K\frac{d^2x}{dt^2} + \frac{dx}{dt} = G(x) + n \tag{4}$$

For an appropriate value of *n*, a negative *n* results in an isolation of the left-half image of the original attractor, whereas a positive *n* results in an isolation of the right-half image of the original attractor.

4. Numerical Results

4.1. New Chaotic Attractors

By using the single-coefficient jerk model described in (1) and (3) based on Fig. 1, six new chaotic attractors are displayed either on an *X-Y* phase plane as shown in Figs. 2(A1), 2(B1), 2(C1), 2(D1), 2(E1) and 2(F1), or on an *X-Z* phase plane as shown in Figs. 2(A2), 2(B2), 2(C2), 2(D2), 2(E2) and 2(F2), respectively. It appears that the new attractors exhibit complex behaviors of chaotic dynamics.

4.2. Compound Structures

For the nonlinearity $G_1(x)$ and $n = -0.09$, a left-half image of the original attractor shown in Figs. 2(A1) and 2(A2) can be isolated as illustrated in Figs. 2(A3) and 2(A4), respectively. In contrast, for $n = 0.09$, another right-half image of Figs. 2(A1) and 2(A2) can be isolated as illustrated in Figs. 2(A5) and 2(A6), respectively. For the nonlinearity $G_2(x)$ and $n = -0.45$, a left-half image of the original attractor shown in Figs. 2(B1) and 2(B2) can be isolated as illustrated in Figs. 2(B3) and 2(B4), respectively. In contrast, for $n = 0.45$, another right-half image of Figs. 2(B1) and 2(B2) can be isolated as illustrated in Figs. 2(B5) and 2(B6), respectively.

For the nonlinearity $G_3(x)$ and $n = -0.78$, a left-half image of the original attractor shown in Figs. 2(C1) and 2(C2) can be isolated as illustrated in Figs. 2(C3) and 2(C4), respectively. In contrast, for $n = 0.78$, another right-half image of Figs. 2(C1) and 2(C2) can be isolated as illustrated in Figs. 2(C5) and 2(C6), respectively. For the nonlinearity $G_4(x)$ and $n = -0.15$, a left-half image of the original attractor shown in Figs. 2(D1) and 2(D2) can be isolated as illustrated in Figs. 2(D3) and 2(D4), respectively. In contrast, for $n = 0.15$, another right-half image of Figs. 2(D1) and 2(D2) can be isolated as illustrated in Figs. 2(D5) and 2(D6), respectively.

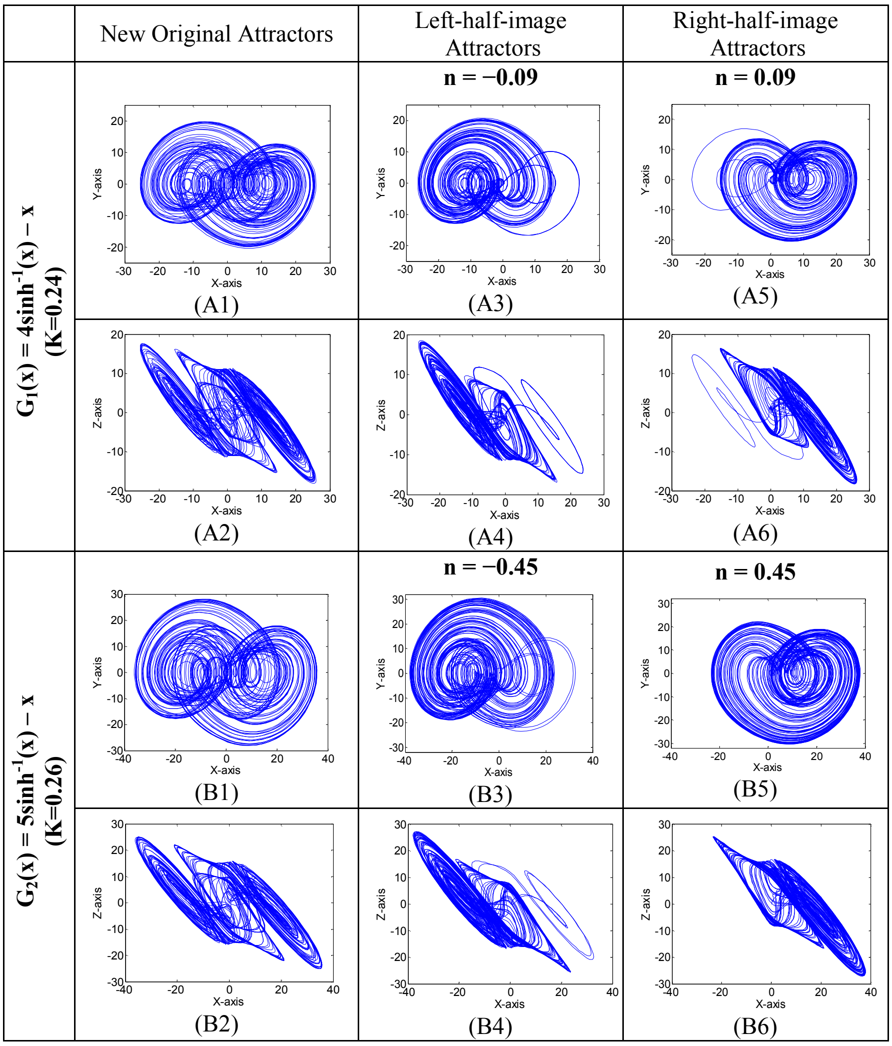

Figure 2. Six new chaotic attractors and the corresponding left- and right-half-image attractors.

For the nonlinearity $G_5(x)$ and $n = -0.21$, a left-half image of the original attractor shown in Figs. 2(E1) and 2(E2) can be isolated as illustrated in Figs. 2(E3) and 2(E4), respectively. In contrast, for $n = 0.21$, another right-half image of Figs. 2(E1) and 2(E2) can be isolated as illustrated in Figs. 2(E5) and 2(E6), respectively. Finally, for the nonlinearity $G_6(x)$ and $n = -0.29$, a left-half image of the original attractor shown in Figs. 2(F1) and 2(F2) can be isolated as illustrated in Figs. 2(F3) and 2(F4), respectively. In contrast, for $n = 0.30$, another right-half image of Figs. 2(F1) and 2(F2) can be isolated as illustrated in Figs. 2(F5) and 2(F6), respectively.

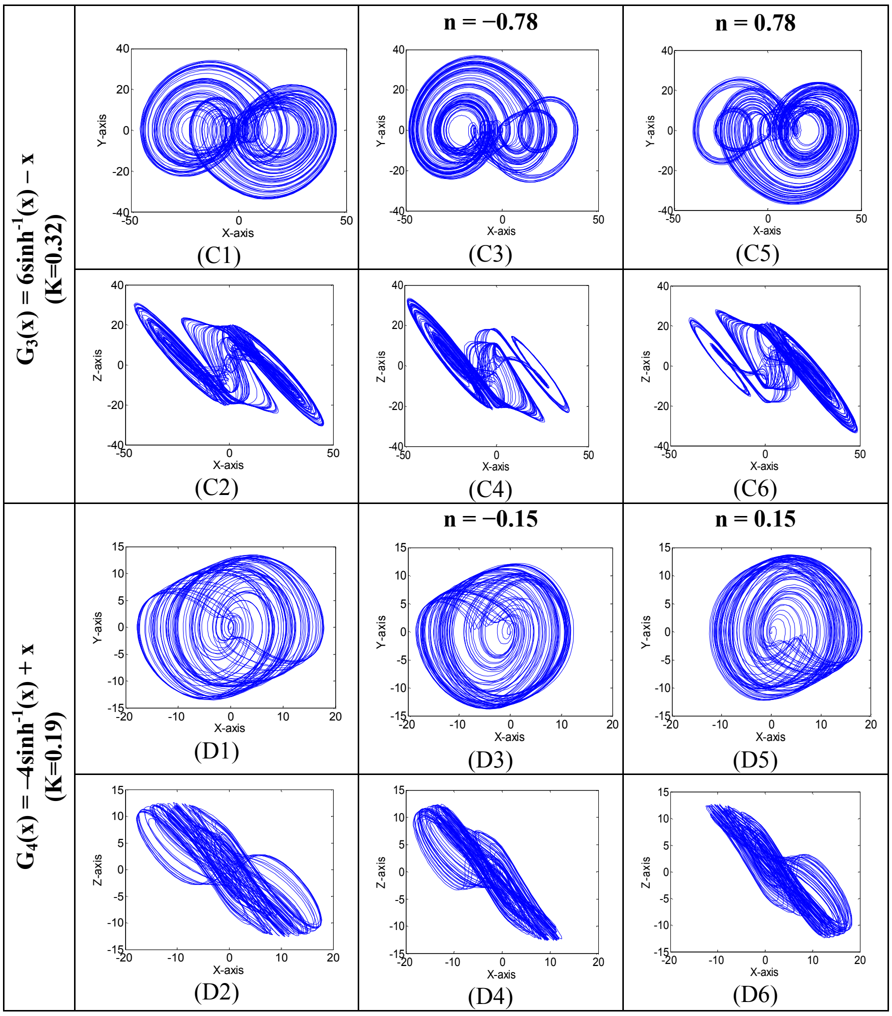

Figure 2. Six new chaotic attractors and the corresponding left- and right-half-image attractors (continued).

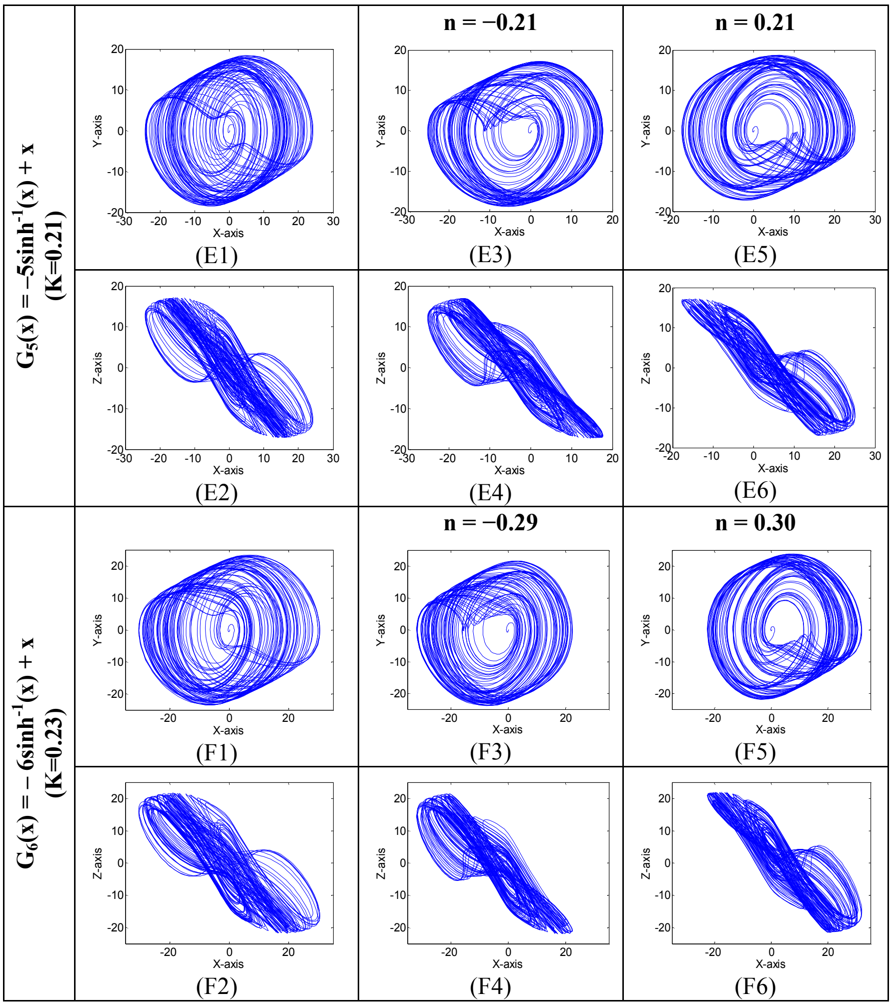

Figure 2. Six new chaotic attractors and the corresponding left- and right-half-image attractors (continued).

5. Conclusions

Six new chaotic attractors in a modified single-coefficient jerk model have been presented through the use of *Sinh*$^{-1}$ nonlinearity and six new values of the single coefficient. In addition, a compound structure of each chaotic attractor has been demonstrated using a half-image operation to obtain either a left- or a right-half-image attractor, each of which can be merged together as a compound structure.

Acknowledgment

This work was supported by Telecommunications Research and Industrial Development Institute (TRIDI), NBTC, Thailand under Grant TARG 2553/002, and the National Research University Project of Thailand, Office of Higher Education Commission.

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