

# Dissipative Solitons in Presence of Quantum Noise

Vladimir L. Kalashnikov

Technical University of Vienna, Institute for Photonics, Gusshausstr. 25-29, A-1040 Vienna, Austria  
(E-mail: [kalashnikov@tuwien.ac.at](mailto:kalashnikov@tuwien.ac.at))

**Abstract.** It is shown, that a dissipative soliton is strongly affected by a quantum noise, which confines its energy scalability. There exists some bifurcation point inside a soliton parametric space, where the energy scalability of dissipative soliton changes drastically so that an asymptotically unlimited accumulation of energy becomes impossible and the so-called “dissipative soliton resonance” disappears.

**Keywords:** Dissipative soliton, Quantum noise, Dissipative soliton resonance.

## 1 Introduction

In the last decade, the concept of a dissipative soliton (DS), that is a strongly localized and stable structure emergent in a nonlinear dissipative system far from the thermodynamic equilibrium was actively developing and became well-established [1]. The unique feature of DS is its capability to accumulate the energy without stability loss [2]. As a result, the DS is energy-scalable. This phenomenon resembles a resonant enhancement of oscillations in environment-coupled systems so that it was proposed to name it as a “dissipative soliton resonance” (DSR) [3]. A capacity of DS to accumulate the energy is of interest for a lot of applications. For instance, it provides the energy scaling of ultra-short laser pulses and brings the high-field physics on table-tops of a mid-level university lab [4].

Nevertheless, the noise properties of DS remain practically unexplored. Such properties promise to be nontrivial because, as was found, the DS can contain the internal perturbation modes, which reveal themselves as the spectrum distortions and the peak power jitter [5]. Moreover, the parametric space of DS and, as a result, the DSR can be modified substantially under action of gain saturation and another dynamic factors [6–8].

In this work, a numerical analysis of DS parametric space taking into account the quantum noise is presented. It is demonstrated, that the noise modifies the DS parametric space substantially and reduces the soliton energy scalability. The different scenarios of DS destabilization are explored. It is found,



that such scenarios are soliton explosion, multipulsing and appearance of rogue DSs. The noise causes a chaotization of multiple soliton complexes so that DS cannot exist above some critical energy level.

## 2 Concept of the DS and the DS parametric space

DS is a strongly localized and stable structure, which develops in a non-equilibrium system and, thus, has a well-organized energy exchange with an environment. This energy exchange forms a non-trivial internal structure of DS, which provides the energy redistribution inside it (e.g., see [1]). In this respect, DS is a primitive analogue of cell.

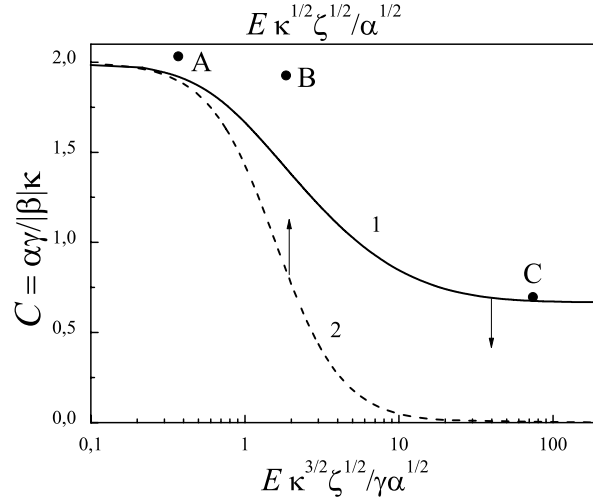
One may think, that a simplest and, simultaneously, sufficiently comprehensive mathematical framework for a DS modeling is provided by the so-called nonlinear Ginzburg-Landau equation (NGLE) [9]. Here, we shall explore the NGLE with the cubic-quintic nonlinearity, which is appropriate, e.g., to modeling of the nonlinear optical and laser systems [10,11]:

$$\frac{\partial a(z, t)}{\partial z} = \left[ -\sigma + (\alpha + i\beta) \frac{\partial^2}{\partial t^2} + (\kappa - i\gamma) |a(z, t)|^2 - \kappa\zeta |a(z, t)|^4 \right] a(z, t). \quad (1)$$

Here,  $a(z, t)$  is a complex “field amplitude” describing the DS profile (e.g., it is a “slowly-varying” field amplitude for an optical DS or an effective “wave function” for a Bose-Einstein (BE) condensate [12]),  $t$  is a “local time” (that is a coordinate along which a DS is localized, e.g., it is a co-moving time-frame for an optical DS or a transverse spatial coordinate for a BE DS),  $z$  is a DS “propagation coordinate” (e.g., it is a number of cavity round-trips for a laser or a time for a BE condensate). The  $\beta$ -coefficient is a group-delay dispersion (GDD) coefficient (or a “kinetic-energy” term for a BE condensate),  $\alpha$  is a squared inverse bandwidth of a spectral filter (e.g., it can be a squared inverse laser gain bandwidth or a “runaway” coefficient for a BE condensate). The  $\gamma$ -coefficient defines a self-phase modulation (SPM) in a nonlinear optical system (a “strength” of three-bosons interaction),  $\kappa$  is a dissipative correction to it (a self-amplitude modulation (SAM) coefficient or a “strength” of boson creation in three-bosons interactions), and  $\zeta$  is a higher-order correction to SAM coefficient. The  $\sigma$ -coefficient is a saturated net-loss coefficient, which defines the energy exchange with an environment (generally speaking, this exchange depends on the DS energy).

Only a sole analytical DS solution for Eq. (1) is known [10] but there are the powerful approximate techniques, which allow exploring the solitonic properties of NGLE [2]. These techniques demonstrate that a DS “lives” in the parametric space with reduced dimensionality. For instance, the DS of Eq. (1) has a two-dimensional parametric space [11] and its representation was called as the “DS master diagram” [2,11,13]. Such a diagram demonstrates some asymptotic corresponding to an infinite DS energy growth  $E \rightarrow \infty$  (e.g.,  $E$  can be associated with an ultrashort pulse laser energy or a mass of BE condensate). This asymptotic was named later as the DSR [3].

The structure of the master diagram is crucial for a DS characterization. The most interesting is the so-called “zero isogain curve”, where  $\sigma \equiv 0$  that corresponds to a “vacuum stability” of Eq. (1) and defines the DS stability border. Such a DS stability border obtained from the adiabatic theory of chirped DS developing in the range of normal GDD ( $\beta > 0$ ) [2] is shown by the solid curve (1) in Fig. 1. The DS is stable below this curve.



**Fig. 1.** Master diagram (parametric space) of DS. Solid curve (1) corresponds to the stability border of chirped DS obtained from the adiabatic theory ( $\beta > 0$ ). For comparison, dashed curve (2) shows the stability border of chirp-free DS obtained on the basis of the variational approximation ( $\beta < 0$ ). The solitons are stable below the corresponding curves. One has note, that the abscissas (i.e. the energy normalizations) differ for two types of solitons (arrows point to the corresponding abscissa). DS evolutions for the parameters corresponding to the points A, B and C are shown in Figs. 2, 3 and 4, respectively.

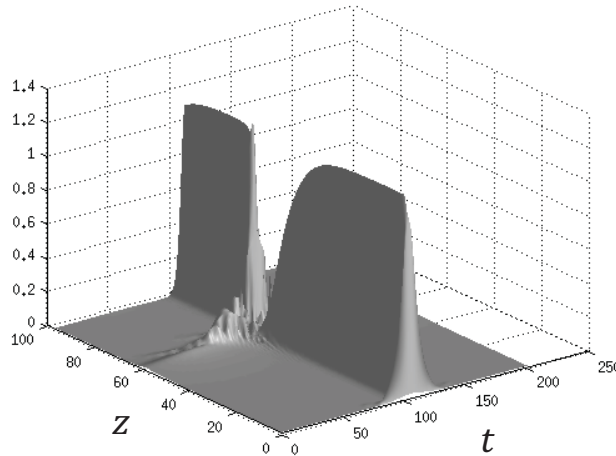
The dimensionless coordinates in Fig. 1 represent a true parametric space of DS and demonstrate the DSR existence for a chirped DS:  $\lim_{C \rightarrow 2/3} E = \infty$ . Physically, the DSR corresponds to a perfect scalability of DS energy that is the DS energy can grow without a change of system parameters (i.e. parameters of Eq. (1)). Of course, the energy inflow is required for such a scaling. This inflow is provided by the energy-dependence of  $\sigma$ -parameter:  $\sigma \approx \xi (E/E' - 1)$  (here  $E'$  corresponds to the energy of a  $t$ -independent solution of Eq. (1);  $\xi$  is a parameter, which is irrelevant for a further consideration) [11].

For comparison, the dashed curve (2) in Fig. 1 shows the stability border for a chirp-free DS obtained on the basis of the variational approximation. The important feature of such a soliton, which develops in the range of anomalous dispersions ( $\beta < 0$ ) is an absence of DSR, so that the energy scaling requires the corresponding scaling of parameters of Eq. (1):  $E \rightarrow \sqrt{5} |\beta| / \zeta \gamma$  for a large  $E$ .

Such a difference in the energy scalability has a simple explanation. The DS of Eq. (1) is power-bounded:  $\max(|a|^2) \leq 1/\zeta$ . Hence, the energy scaling can be provided by only soliton stretching. But, from the area theorem for a soliton of the nonlinear Schrödinger equation, such a stretching requires the GDD growth or the SPM reduction:  $E = 2\sqrt{2}|\beta|/\zeta\gamma$ . On the other hand, for the chirped DS, such a stretching results from the chirp growth so that manipulations with the parameters of Eq. (1) are not necessary for an asymptotical energy growth. At the same time, a stretching of chirped DS is reversible due to a posterior chirp-compensation so that the minimum soliton width is defined by the energy-independent value of  $\sqrt{3\alpha/2}$ .

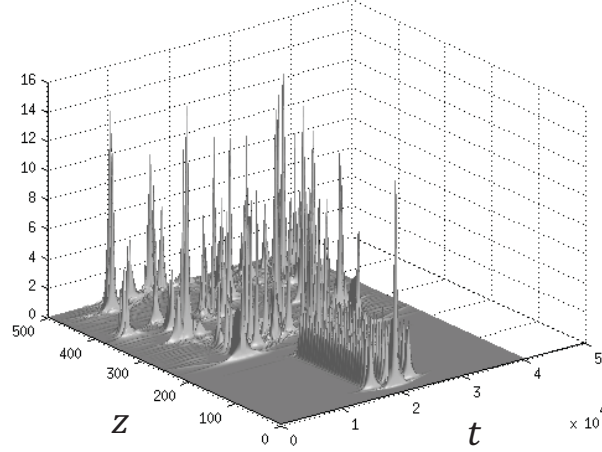
### 3 Scenarios of DS destabilization

Numerical simulations of Eq. (1) reveal three main scenarios of chirped DS destabilization. The first one corresponding to low energies (point *A* in Fig. 1) is the so-called soliton explosion (Fig. 2) [14]. The explosion results from the interaction with slowly growing vacuum perturbations causing aperiodic destruction of DS with a subsequent its recreation. Vacuum is unstable in this case ( $\sigma < 0$ ).



**Fig. 2.** Explosive chirped DS ( $|a(z, t)|^2$ -profile) corresponding to the point *A* in Fig. 1. The propagation coordinate  $z$  and the local time  $t$  are given in arbitrary units.

In a middle range of DS energies (point *B* in Fig. 1), a very interesting regime of destabilization appears (Fig. 3). There are the rogues DSs [15]. In this regime, there exists some localized complex of strongly and chaotically interacting, decaying and emerging solitons. The peaks appearing in such a complex can exceed substantially the mean power and the statistics of their appearance is not Gaussian. Vacuum is unstable in this case, as well.



**Fig. 3.** Rogue chirped DSs ( $|a(z, t)|^2$ -profile) corresponding to the point  $B$  in Fig. 1.

At last, the most typical scenario of a high-energy DS destabilization (point  $C$  in Fig. 1) is the multipulsing (Fig. 4). This regime corresponds to a generation of several stable solitons, which can be bounded within multisoliton complexes. Such a regime is typical also for the chirp-free DSs. The main mechanism causing multipulsing is the growth of spectral dissipation that decreases the DS energy [16]. As a result,  $\sigma$  parameter becomes negative that destabilizes vacuum and the new solitons develop. After formation of several additional solitons with the reduced spectral widths, the spectral dissipation decreases and the vacuum becomes stable again. Under some conditions, the strong interactions between DSs inside a complex can result in strongly unsteady dynamics including formation of rogue DSs.

#### 4 Chirped DS under the noise action

The quantum noise can be included in Eq. (1) in the form of an additive complex stochastic term  $\psi(z, t)$  with the correlation:

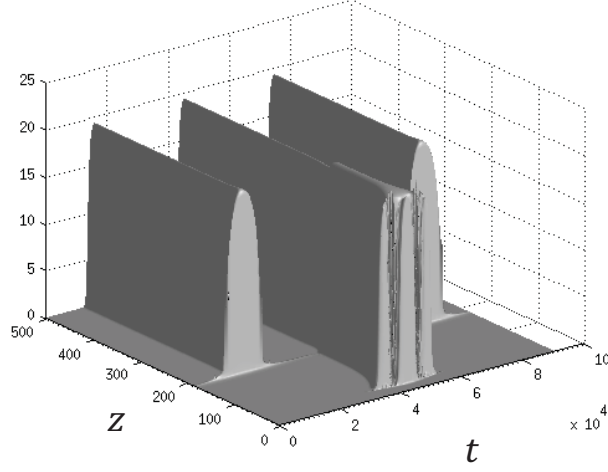
$$\langle \psi(z, t) \psi^*(z', t') \rangle = \Gamma \delta(z - z') \delta(t - t'),$$

where  $\Gamma$  describes the noise “power”. For the spontaneous noise in a laser gain medium, one has [17]:

$$\Gamma = 2\sigma\theta \frac{h\nu}{\Delta t},$$

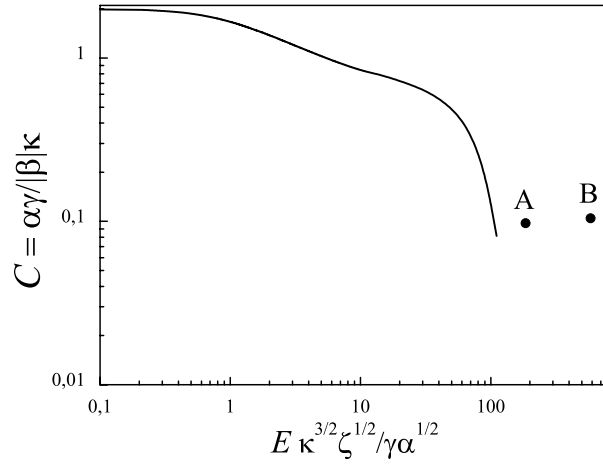
where  $\theta$  is the enhancement factor due to incomplete inversion in an active medium,  $\Delta t$  is the time step in subdividing of time window representing  $a(t)$ .

The inclusion of such a term in Eq. (1) transforms the master diagram drastically. Solid curve in Fig. 5 demonstrates the DS stability border in this case. Its noiseless analog is the solid curve 1 in Fig. 1. One can see, that the DS

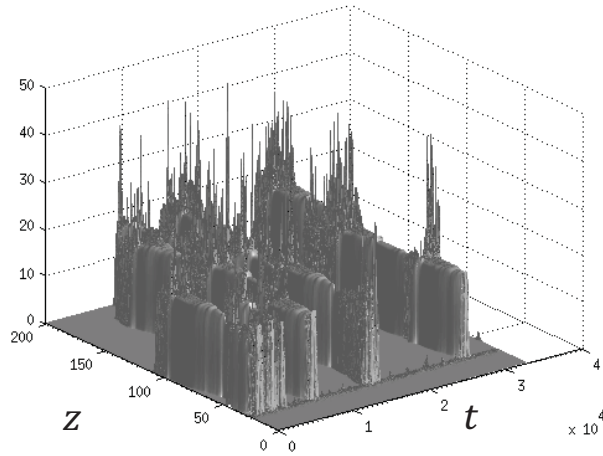


**Fig. 4.** Multi-DS complex ( $|a(z,t)|^2$ -profile) corresponding to the point  $C$  in Fig. 1.

stability conditions change after some bifurcation point ( $E\kappa^{3/2}\zeta^{1/2}/\gamma\alpha^{1/2} \approx 20$  in our case) so that the energy scaling needs a substantial decrease of the  $C$ -parameter. For a mode-locked laser, this corresponds to a substantial GDD-growth or/and a SPM-reduction required for the DS stabilization. Thus, the DSR disappears under the noise action.



**Fig. 5.** Master diagram of the chirped DS in the presence of quantum noise.  $\Gamma = 10^{-10}/\gamma$ . DS evolutions for the parameters corresponding to the points  $A$  and  $B$  are shown in Figs. 6 and 7, respectively.



**Fig. 6.** Chaotic multi-DS complex ( $|a(z, t)|^2$ -profile) corresponding to the point  $A$  in Fig. 5.

Moreover, the DS becomes completely unstable above some critical energy ( $E\kappa^{3/2}\zeta^{1/2}/\gamma\alpha^{1/2} \approx 100$  in our case). Only completely chaotic regimes exist starting from this limit.

Fig. 6 demonstrates, that the multipulsing regime in high-energy limit with noise becomes completely chaotic with the elements of rogue soliton dynamics.

A further energy growth enhances chaotization (Fig. 7) so that eventually the DS becomes completely “dissolved” in a sea of amplified noise.

Another important feature of a high-energy DS in the presence of noise is that the soliton emergence is random, that is it depends on both a random sample of initial noise conditions and their evolution. Thus, the stability border for a high-energy DS becomes “fuzzy”.

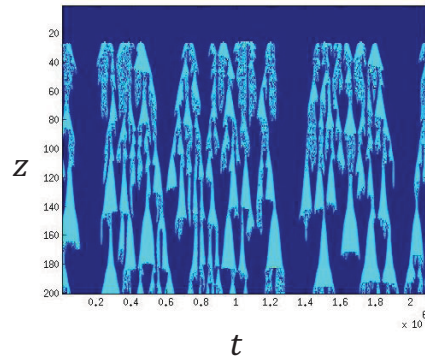
## 5 Conclusion

The numerical analysis of NGLE has demonstrated that the main scenarios of chirped DS destabilization are i) exploding instability for low soliton energies, ii) rogue soliton generation for middle- and high energy levels, and iii) multipulsing. It was found, that the energy scalability of chirped DS is affected strongly by quantum noise so that a noise destroys the DSR and the soliton energy scaling requires a substantial GDD increase. Starting from some energy level, a noise prevents the DS formation at all so that a zoo of chaotic regimes appears. This confines a reachable maximum of DS energy.

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**Fig. 7.** Enhanced chaoticization of the multi-DS complex (contour-plot of  $|a(z, t)|^2$ -profile) corresponding to the point  $B$  in Fig. 5.

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