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Chaos in Digital Currency Markets

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Abstract: Bitcoin was introduced in 2009 as an open source, peer-to-peer payment network for sending payments using a client software. Bitcoin has lower transaction fees compared to credit card processors which makes it a preferable transaction agency for merchants. In this paper, daily exchange prices of bitcoin in the years of 2011 to 2014 for different currencies have been analyzed using nonlinear time series analysis techniques. To apply the analysis, phase space is reconstructed by using delay time obtained from mutual information and autocorrelation of data with an embedding dimension suggested by the false nearest neighbors method. Calculated positive Lyapunov exponents indicate a possible chaotic behavior.
Keywords: Chaotic modeling, Time series analysis, Mutual information, Embedding dimension,Dynamical systems.

1. Introduction

Bitcoin's main purpose is to create a payment mode which is not controlled by any agency (by bank, government etc.). They are not printed, like dollars or euros. They are produced by lots of people running computers all around the world. Currently, twenty five new Bitcoins are released with each block every ten minutes. However, this will be halved to 12.5 BTC (Bitcoin unit) during the year 2017 and halved continuously every four years after until a hard limit of twenty one million Bitcoins is reached. An algorithm that becomes exponentially more difficult over time controls the rate of supply.
In this paper we involve time series analysis of experimental data from the average daily currency data of Bitcoin in the international software market. For now, we examine USD and Euro parity of Bitcoin in given time interval.

2. Theory and application

We must first examine the delay time in order to construct the phase space. If the time delay is taken too short then components of the reconstructed vectors will be close to each other, causing the state space picture to appear on the diagonal line, therefore we will have loss of information about the real system. On the other hand using a too long delay time will cause the correlations between the components of reconstructed vectors to be lost and signals will be mistakenly recognized as random. Information between a random variable and another random variable called mutual information. We can only observe our

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sending information to a channel by getting back the information coming from that channel.

For example, let X and Y be random variables with common distribution p(X,Y). The joint probability of observing x by a measurement of X and Observing y by a measurement of Y, $P_{xy}(x;y)$, should be different from the product of the individual probabilities of measuring x and y out of the sets X and Y respectively, P(x) and P(y) if there is correlation between the two sets. The logarithm of that ratio in bits is therefore called the average mutual information of X and Y given by $\log_2 \frac{P_{XY}(x\,;y)}{P(x)P(y\,)}$. The weighted average of the average mutual information is given by the following Formula;

$$I(X;Y)=-\sum_x \sum_y P_{XY}(x;y\,)\log_2 \frac{P_{XY}(x;y)}{P(x)P(y)}$$

To apply this formula to time series analysis, we assume that S(n) is Set X and S(n+t) is Set Y. Here we obtain the average mutual information as follows:

$$I(t)=-\sum_x \sum_y P(s(n+t),s(n))\log_2\left[\frac{P(s(n+t),s(n))}{P(s(n+t))P(s(n))}\right]$$

$$I(X:Y)=D(p(x,y)\|p(x)p(y))$$

$$I(X;Y)=\sum_x\sum_y p(x,y)\log\frac{p(x,y)}{p(x)p(y)}$$

Where p(x) and p(y) are the probability distributions and the entropy is the distance between actual distribution and the distribution where the mutual informations are equal. Figure 1 shows parity prizes between 2011 and 2014 taken from bitcoincharts.com.

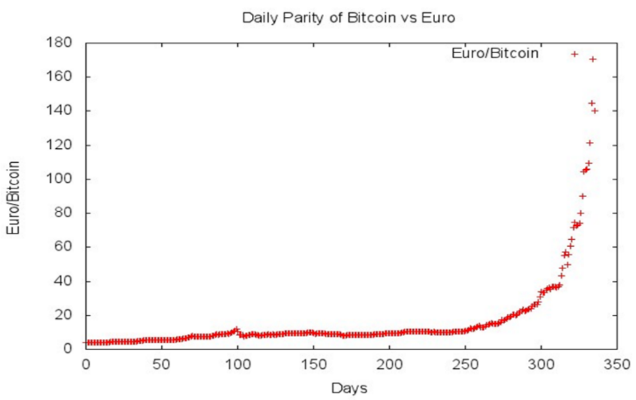

Fig. 1. Daily parity of Euro versus Bitcoin

Then by using the TISEAN programming tool package, we find the mutual information as shown in Figure 2.

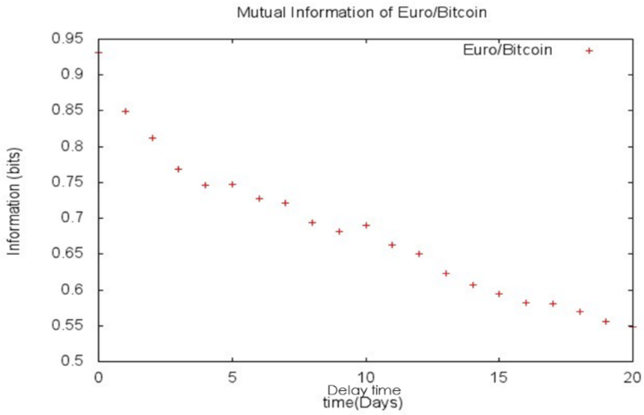

Fig. 2. Mutual information of parity Euro versus Bitcoin

To estimate the delay time we find the minimum of the mutual information which is four. The delay time is needed for finding the embedding dimension. If the embedding dimension is less than the actual dimension, points which are not neighbors on the original attractor fall into same neighborhood. Then finding false-nearest neighbors of all points on embedded attractor is necessary. We can show time delayed vector of nearest neighbors and distance in d+1 dimension of between two neighbors can be calculated like this;

$$\vec{y}_{NN}(k) = \left[S_{NN}(k), S_{NN}(k+\tau), ..., S_{NN}(k+(d-1)\tau)\right]$$

$$\left[R_{d+1}(k)\right]^2 = \left[\sum_{m=1}^{d+1}\left[S(k+(m-1)\tau) - S_{NN}(k+(m-1)\tau\right]^2\right]$$ $R_{d+1}(k)$ can be expressed in terms of $R_d(k)$ as

$(R_{d+1}(k))^2=(R_d(k))^2+(S_{NN}(k+(m)t)-S(k+(m)t))^2$

From this result we can define a threshold value $R_T(k)$ using

$$\frac{(S_{NN}(k+(m)t) - S(k+(m)t))}{R_m(k)} \rangle R_T(k)$$

Note that we are embedding a one dimensional signal originating from a system with arbitrarily dimensionality, hence we use this criterion to find the appropriate embedding dimension in the delay vector, by looking at the point where the number of false neighbors stabilize.

Chaoticity is expressed in terms of Lyapunov exponents. To calculate this, we start with two nearby trajectories separated by a distance |x(0)|. At time t, separation is δx(t)=δx(0) exp(μt).

Average of μ, the rate of exponential separation over the trajectory is the Lyapunov Exponent. In a N dimensional system, we can change δx(0) in N independent ways so that there are N Lyapunov exponents.

The Lyapunov Exponent is calculated as follows, First, find a reference point S_{n0} and let U be a hyper sphere in a distance ε.

$$S(\Delta n) = \frac{1}{N}\sum_{n0=1}^{N} \ln\left[\frac{1}{|U(S_{n0})|}\sum_{S_n=\bigcup(S_{n0})} \left|\left[S_{n0+\Delta n} - S_{n+\Delta n}\right]\right|\right]$$

If ε is too small, we can not find neighbors, if it is too large, a periodic component may be missed. Second,for different values of ε, if the graph of S(Δn) vs Δn has positive slope, we have positive Lyapunov exponent. Positive Lypaunov exponent means that there is a chaos.

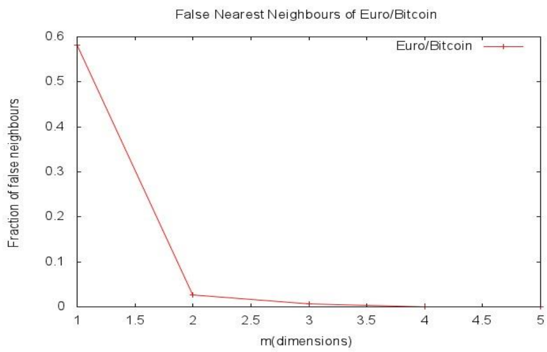

Fig. 3. False Nearest neighbors of Euro/bitcoin

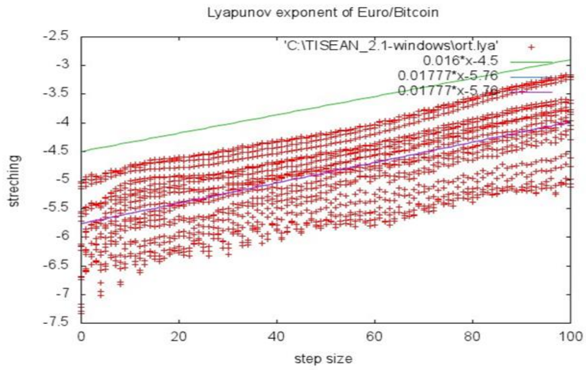

Fig. 4. Lyapunov exponent of Euro/bitcoin

As you can see in Fig. 4. Lyapunov exponent shows a positive increase therefore, we can conclude chaotic behavior. The same analysis is made for US Dollar parity and we can again see a positive Lyapunov exponent

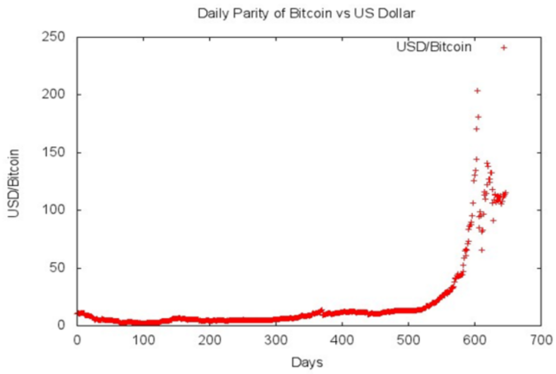

Fig. 5. Daily parity of US Dollar vs Bitcoin

In Fig. 5. Daily parity of dollar is shown and mutual information and false nearest neighbors are observed and finally positive Lyapunov exponent is observed in Fig. 6.

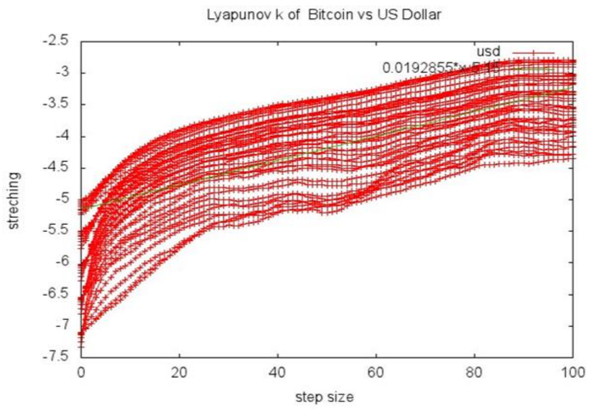

Fig. 6. Lyapunov exponent of US Dollar/Bitcoin

With the globalization and last decades, financial markets have become more sensitive and interactive. Furthermore, digital currency becomes important role for financial market because of its better purposes. In this study, the Bitcoin parity has been analyzed through chaos theory and nonlinear time series to investigate whether bitcoin also follows typical market trends these methods really determines market behavior. The results of the analyses show that methods based on chaos theory explain the financial data better than classical time series analyses methods.

3. Conclusions

Daily average currency flow data from USD and Euro parity of Bitcoin are examined. The data expands through the years nearly 2011-2014. The values obtained for delay time, embedding dimension and maximal Lyapunov exponents are shown. Calculated positive maximal Lyapunov exponent indicates a possible chaotic behavior for the Bitcoin currency dynamics. A further study can be on the dimension of the attractor will reveal more information about the chaotic behavior of the digital currency.

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