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Reconstruction of Evaporation Dynamics from Time Series

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Abstract: The maximum amount of water loses from the reservoirs take place through evaporation. Thus it is important to know the dynamical system that governs the evaporation process. In this study, the Trajectory Method has been applied in order to obtain the differential equation from reconstructed phase space using evaporation time series. The trajectory method has been successfully applied in order to obtain the dynamical system that represents the periodic behavior of evaporation process.

Keywords: Dynamical system, Trajectory Method, Ordinary Differential Equations, Water Losses, Evaporation

1. Introduction

Water is the most vital substance for sustainability of life on planet earth. Unfortunately its distribution on earth both in time and in space is not uniform. This means that the water problem existed in the past, exists today and will exist in the future. On the other hand, especially in recent years water problem has gained much importance due to climate change. The state of the art climate models have shown that water related problems will be experienced more frequently in the future. This worsens the water related problems to a great extent. Thus it is mandatory to make intensive researches on the water resources and managements. In this context, water loses from all kind of water reservoirs are very important to be brought to a minimum level. As known well, the maximum amount of water loses from the reservoirs take place through evaporation. Thus it is important to know the dynamical system that governs

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the evaporation process. In this study, the Trajectory Method has been applied to reconstruction of differential equation that governs the behavior of evaporation process. The brief history of the trajectory method used in this study is as follows. Crutchfield and McNamara (1986) have made some important attempt to reconstruct the differential equation from time series. These two researchers have suggested two approximations about the issue. The first of them is the determination of local dynamic that considers the short-term behavior of the system while the second approach deals with the dynamic of the whole attractor that consider the long-term behavior of the system. Almost at the same time with the aforementioned studies, Cremers and Hübler (1986) have developed the flow method that considers the sort-term behavior of the system. The flow method is applied to all points on the attractor. Thus it does not consider the long-term behavior of the system dynamic. Then Breeden and Hübler have developed this approach to include all of the system variables that could not be observed. In the end, Eisenhammer et al. (1991) have combine both short and long-term behavior of the system and they called their approach "trajectory method". In this study, the trajectory method has been successfully applied in order to obtain the dynamical system of evaporation process.

2. Trajectory Method

Trajectory method is based on the reconstruction of differential equations which produce the trajectory resembling the original trajectory. In other word, the reconstructed model is the best possible model reflecting the original model (Perona et al., 2000).

A set first order ordinary differential equations can be given as

$$\dot{x} = f(x,t) \tag{1}$$

where x and t represent the variable vector and time, respectively. To reconstruct the equation of motion it is necessary to obtain the differential equations of model trajectory as close as possible to the original trajectory. On the other hand, mathematical form of the model should be determined ab initio.

According to theory of dynamical system, time evolution of a system can be given by its trajectories in a phase space. Coordinates of this space are formed by state variables which are necessary to reflect the time evolution of the system under study. Every trajectory in this space represents the different time evolution of the system that corresponds to different initial conditions. Phase portraits have distinct patterns that attract all trajectories. This type of a pattern is called attractor. All initial conditions of which trajectories captured from the attractor defines a domain of attraction. Systems that show deterministic evolution have low dimensional attractors like point, limit cycle and torus. These kinds of attractors can be characterized by an integer dimension. An important property of these kinds of attractors is that trajectories that converge onto them remain in a fixed distance from each other. This property ensures the system to be predictable for a long period of time (Koçak, 1996).

It is possible to reconstruct the phase space from a time series of one state variable sampled at regular time intervals Δt. For this to be done, some information and topological properties (e.g. dimension) of the attractor should be first estimated from the time series. Dimension of an attractor is the number of variable necessary to define the dynamics of the underlying system.

Packard et al., (1980) have suggested the reconstruction of phase space in order to obtain some invariant measures from an observed turbulent or chaotic flow. This can be achieved via transformation of the dynamical process to a higher dimensional space (embedding) by adding an extra independent dimension until no further information gain is impossible. One of these coordinates is formed by the time series itself and the remaining independent coordinates are formed by derivatives of the time series up to $(m\text{-}1)^{\text{th}}$ order. As a result, phase portrait of time evolution of a dynamical system can be represented in a new m-dimensional space spanned by a single state variable and its successive derivatives.

In this study, phase space is reconstructed from univariate or single time series (evaporation). Thus it is necessary to mention briefly from phase space reconstruction. Let's take a time series given as

$$x_i \in R, \quad i = 1, 2, \ldots, N. \tag{2}$$

Then the reconstruction procedure is given as

$$\begin{gathered} X_i = (x_i, x_{i-\tau}, \ldots, x_{i-(m-1)\tau}) \in R^m \\ i = 1 + (m-1)\tau, 2 + (m-1)\tau, \ldots, N-1, N \end{gathered} \tag{3}$$

where X_i is an m-dimensional vector.

This pseudo-phase space preserves the structure of the attractor embedded in the original phase space, (Takens, 1981). In Eq (3) τ is called time delay and should be calculated from time series by using autocorrelation function or mutual information function. Differential equation used in the trajectory method is assumed in the following form:

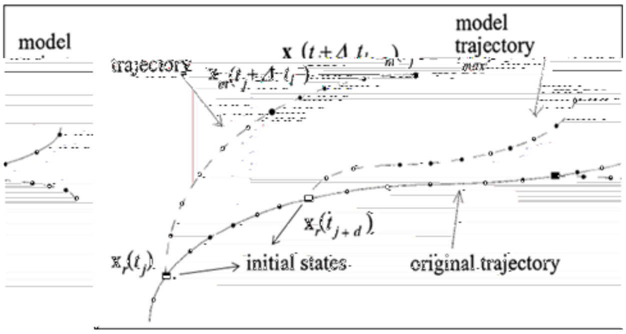
$$\dot{x}_i = \sum_{k=1}^{K} c_{i,k} F_{i,k}(x_1, x_2, \ldots, x_D) \qquad i = 1, 2, \ldots, D \tag{4}$$

where $c_{i,k}$s are coefficients of differential equation and $F_{i,k}(x_1, x_2, \ldots, x_D)$s are approximating functions. On the other hand K and D represent the number of approximating function and state variable, respectively. If $F_{i,k}$ is chosen as the 3[rd] degree polynomial then Eq (4) can given as

(5)

The trajectory method is very effective way of representing both short and long term behavior of dynamical system in the space of K functions.

Figure 1 outlines the trajectory method. As shown in this figure, model (Eq (4)) is run with the initial conditions (j=1,2,…,j_{max}) chosen along the original trajectory ($x_r(t_n)$, n=1,2,…,N).



same data point. On the other, hand l_{max} determines how many steps the model will be run in order to catch both sort and long-term behavior of the system. In other words, l_{max} is the number of points used for comparison between the single reconstructed trajectory and the original trajectory, starting from the initial state set on the latter. Δt_l in Eq (6) is the time interval between the integration steps of the model equation. This quantity can be calculated as

$$\Delta t_l = h(2^{l-1}) \tag{7}$$

where h is the interval between the observations or integration step in case of numerical integration. The optimum value of $c_{i,k}$ are obtained by minimization the quality function Q.

$$Q_{\min} = \min_{C_{i,k}} Q \qquad (i = 1,2,\ldots,D;\ k = 1,2,\ldots K) \tag{8}$$

Eq (6) can be stated as given below

$$Q = \sum_{j=1}^{j_{\max}} \sum_{l=1}^{l_{\max}} \sqrt{\sum_{i=1}^{D} \left[\left(\int_{t_j}^{t_j+\Delta t_l} \dot{x}_{m_i}(\tau)d\tau \right) + x_{m_i}(t_j) - x_{r_i}(t_j+\Delta t_l) \right]^2} \tag{9}$$

The integral given in Eq (9) represents the change of $x_{m_i}(t)$ between the time interval $[t_j, t_j+\Delta t_l]$ and can be stated as

$$\int_{t_j}^{t_j+\Delta t_l} \dot{x}_{m_i}(\tau)d\tau = x_{m_i}(t_j+\Delta t_l) - x_{m_i}(t_j) = c_{i,1} \int_{t_j}^{t_j+\Delta t_l} F_{i,1}(\tau)d\tau + \ldots + c_{i,k} \int_{t_j}^{t_j+\Delta t_l} F_{i,k}(\tau)d\tau \tag{10}$$

The integrals in Eq (10) should be calculated numerically because the functions $F_{i,k}$ are all unknown functions. If the partial derivative of Q with respect to unknown coefficients $c_{i,k}$ is set to zero, then the following set of linear equation is obtained:

—— „ A2447.190 1 12,344 Tf1,3 0 (11)
BT 27 ET Bz 0 Tc[(1)] TJ ET BT 6 0 05 B

The matrix $A_{k,z}^{(i)}$ and the vector $B_k^{(i)}$ are as given in Eqs (12) and (13), respectively.

$$A_{k,z}^{(i)} = \sum_{j=1}^{j_{\max}} \sum_{l=1}^{l_{\max}} \left[\left(\int_{t_j}^{t_j+\Delta t_l} F_{i,k}(\tau) d\tau \right) \left(\int_{t_j}^{t_j+\Delta t_l} F_{i,z}(\tau) d\tau \right) \right] \quad (12)$$

$$B_k^{(i)} = \sum_{j=1}^{j_{max}} \sum_{l=1}^{l_{max}} \left[\left(x_{r_i}(t_j + \Delta t_l) - x_{m_i}(t_j) \right) \left(\int_{t_j}^{t_j+\Delta t_l} F_{i,k}(\tau) d\tau \right) \right] \quad (13)$$

The matrix *A* is reversible. By solving Eq (11) a new set of coefficients $c_{i,k}$ are obtained then these coefficients are used in the next optimization cycle. This process continues until the optimum values of coefficients are obtained (Perona et al., 2000).

3. Application to Evaporation Data

Daily evaporation totals used in this study are observed in the Ercan Meteorology Station located in North Cyprus. Observation period covers 2001-2010; total number of data points is 3652. In this study, before the application of the trajectory method, the original time series smoothed out by using loess method (Cleveland, 1979). Figure 2 shows the original and the smoothed out time series together.

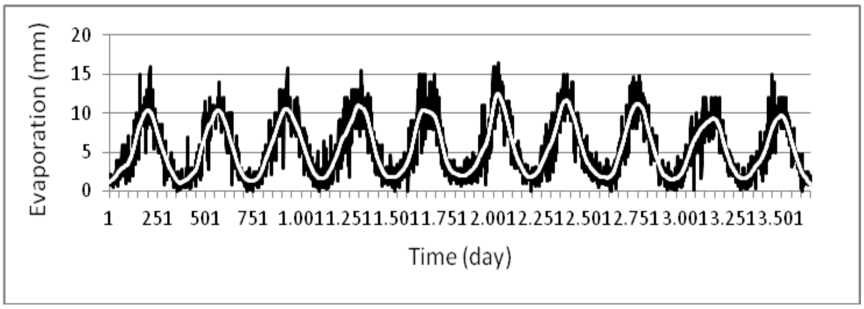


Figure 2. Evaporation time series (black) and smoothed out series (white).

By using smoothed time series phase space is reconstructed. As mentioned before, for phase space reconstruction two parameters namely time delay and embedding dimension are necessary. The time delay is determined by using Mutual Information Function (MIF) approach (Fraser, 1986). The first minimum value is taken as the optimum time delay (see Figure 3). As seen from Figure 3, the first minimum of the MIF is τ=112. On the other hand embedding dimension is assumed m=3.

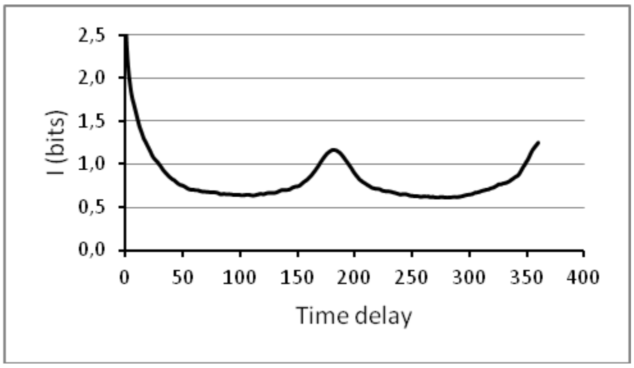


Figure 3. Mutual information of smoothed evaporation time series.

The phase space of evaporation process is reconstructed by taking time delay 112 and embedding dimension 3. Projection of the resulting attractor onto 2-dimension is given in Figure 4. As depicted in this figure smoothed attractor shows almost quasi-periodic behavior. Put another way, the behavior of this attractor in phase space is neither periodic nor aperiodic. This result shows that it will be reasonable to model the periodic structure or limit cycle of this attractor. The trajectory model has been applied to smoothed evaporation time series. The resulting limit cycle is given in Figure 5. As shown from this figure starting from an initial condition, the trajectory eventually converge the stable periodic orbit.

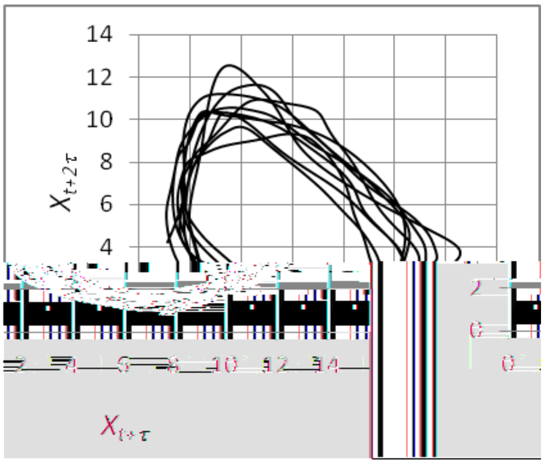


Figure 4. Projection of the attractor onto plane.

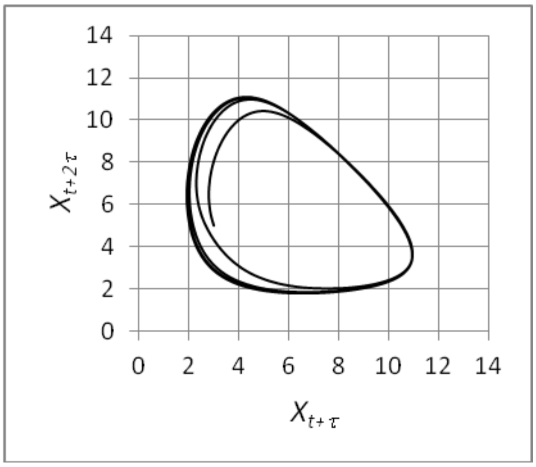


Figure 5. Periodic attractor of evaporation process.

4. Results and Discussion

Water reservoirs are very important in producing hydraulic energy, irrigation, flood control, drinking water, recreational purposes, etc. On the other hand there are some water loses from water reservoirs. The most important water loses take place by evaporation process. Thus, it is important to know the main dynamic of the evaporation.

In this study the trajectory method, the state art of the inverse problem solving method, is applied to evaporation process. Other variables that affect the evaporation such as temperature, wind speed, relative humidity, solar radiation, etc. are not considered in this application. In other words phase space reconstruction from univariate time series is used instead of multivariate approach. After the reconstruction process, the trajectory method is applied to smoothed evaporation data. The limit cycle or periodic behavior of the evaporation has been successfully reconstructed in the form of a set of differential equation which has three state variables.

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