# Mutual information rate and topological order in networks

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**Abstract.** In this paper we study the evolution of the information flow associated with a topological order in networks. The amount of information produced by a network may be measure by the mutual information rate. This measure and the synchronization interval are expressed in terms of the transversal Lyapunov exponents. The networks are constructed by successively joining one edge, maintaining the same number of nodes, and the topological order is described by the monotonicity of the network topological entropy. The network topological entropy measures the complexity of the network topology and it is expressed by the Perron value of the adjacency matrix. We conclude that, as larger the network topological entropy, the larger is the rate with which information is exchanged between nodes of such networks. To illustrate our ideas we present numerical simulations for several networks with a topological order established.

**Keywords:** Mutual information rate, topological entropy, networks.

## 1 Introduction and motivation

Information theory is an area of mathematics and engineering, concerning the quantification of information and it benefits of matters like mathematics, statistics, computer science, physics, neurobiology and electrical engineering. Information theory and synchronization are directly related in a network. The entropy is a fundamental measure of information content and the topological entropy can describe the character of complexity of a network, see for example [10]. In [6], using the mutual information rate to measure the information flow, we have proved that the larger the synchronization is, the larger is the rate with which information is exchanged between nodes in the network. Although the important growth in the field of complex networks, it is still not clear which conditions for synchronization implies information transmission and it is still not known which topology is suitable for the flowing of information.

Received: 7 March 2013 / Accepted: 12 September 2013

ISSN 2241-0503

Nevertheless, we conclude with this study that, the more complex is a network, expressed by its topological entropy, the larger is the flux of information.

In this work we study the relationship between the topological order in networks and the transmission of information. The topological order in networks is described by the monotonicity of the network topological entropy. The concept of the network topological entropy was previously introduced in [10]. However, there are several concepts of network entropy, see [7]. We will use the one based on symbolic dynamics. In Sec.2, we present some preliminaries concepts to be used in the following, such as: fundamental notions of graphs theory, conditions for complete synchronization, comunication channel and mutual information rate. Sec.3 is devoted to the study of topological order in networks, using the definition of the network topological entropy. In Sec.4, numerical simulations are presented for several networks with a topological order established. Finally, in Sec.5, we discuss our study and provide some relevant conclusions.

## 2 Preliminaries concepts

In this section, we introduce some notions and basic results on graphs and networks theory. Mathematically, networks are described by graphs (directed or undirected) and the theory of dynamical networks is a combination of graph theory and nonlinear dynamics. From the point of view of dynamical systems, we have a global dynamical system emerging from the interactions between the local dynamics of the individual elements. The tool of graph theory allows us to analyze the coupling structure between them.

A graph $G$ is an ordered pair $G = (V, E)$, where $V$ is a nonempty set of $N$ vertices or nodes $v_i$ and $E$ is a set of edges or links, $e_{ij}$, that connect two vertices $v_i$ and $v_j$. We will only consider the case of undirected graphs, that means that the edge $e_{ij}$ is the same as the edge $e_{ji}$. If the graph $G$ is not weighted, the adjacency matrix $A = A(G) = [a_{ij}]$ is defined as follows:

$$a_{ij} = \begin{cases} 1, & \text{if } v_i \text{ and } v_j \text{ are connected} \\ 0, & \text{if } v_i \text{ and } v_j \text{ are not connected} \end{cases} .$$

The degree of a node $v_i$ is the number of edges incident on it and is denoted by $k_i$. For more details in graph theory see [4].

Consider a network of $N$ identical chaotic dynamical oscillators, described by a connected and undirected graph, with no loops and no multiple edges. In each node the dynamics of the oscillators is defined by $\dot{x}_i = f(x_i)$, with $f : \mathbb{R}^n \to \mathbb{R}^n$ and $x_i \in \mathbb{R}^n$ is the state variables of the node $i$. The state equations of this network are

$$\dot{x}_i = f(x_i) + \sigma \sum_{j=1}^{N} l_{ij}\ x_j, \quad \text{with} \quad i = 1, 2, ..., N \tag{1}$$

where $\sigma > 0$ is the coupling parameter, $L = [l_{ij}] = A - D$ is the Laplacian matrix or coupling configuration of the network. One of the most important subjects under investigation is the network synchronizability. It may be studied fixing the connection topology and varying the local dynamics or fixing the local dynamic and varying the connection topology [5]. In [9] it was establish that complete synchronization can be achieved provided that all the conditional Lyapunov exponents are negative. In Sec.4, we use this result to determine the synchronization interval. The negativity of the conditional Lyapunov exponents is a necessary condition for the stability of the synchronized state, [3]. It is also a mathematical expression of the decreasing to zero of the logarithm average of the distance of the solutions on the transverse manifold to the solutions on the synchronization manifold.

A communication channel represents a pathway through which information is exchanged. In this work, a communication channel is considered to be formed by a transmitter $S_i$ and a receiver $S_j$, where the information about the transmitter can be measured. In a network, each one of the links between them, i.e., each one of the edges of the corresponding graph, represents a communication channel. In [1], it is defined $I_C(S_i, S_j)$, the mutual information rate (MIR) between one transmitter $S_i$ and one receiver $S_j$, by

$$I_C(S_i, S_j) = \begin{cases} \lambda_{\|} - \lambda_{\perp} \text{ , if } \ \lambda_{\perp} > 0 \\ \\ \lambda_{\|} \qquad\quad \text{ , if } \ \lambda_{\perp} \leq 0 \end{cases}, \tag{2}$$

where $\lambda_{\|}$ denotes the positive Lyapunov exponents associated to the synchronization manifold and $\lambda_{\perp}$ denotes the positive Lyapunov exponents associated to the transversal manifold, between $S_i$ and $S_j$. $\lambda_{\|}$ represents the information (entropy production per time unit) produced by the synchronous trajectories and corresponds to the amount of information transmitted. On the other hand, $\lambda_{\perp}$ represents the information produced by the nonsynchronous trajectories and corresponds to the information lost in the transmission, the information that is erroneously retrieved in the receiver. For more details and references see for example [1] and [2]. In [6], we prove that, as the coupling parameter increases, the mutual information rate increases to a maximum in the synchronization interval and then decreases.

## 3 Topological order in networks

In this section we study a topological order in networks, which are constructed by successively joining one edge, maintaining the same number of nodes. This topological order is described by the monotonicity of the network topological entropy. The introduction of the network topological entropy concept was made in [10], which requires a strict and long construction, using tools of symbolic dynamics and algebraic graph theory. However, we present some basic aspects of this definition. The topological entropy $h_{top}(X)$ of a shift dynamical system $(X, \sigma)$ over some finite alphabet $\mathcal{A}$ is defined by

$$h_{top}(X) = \lim_{n\to\infty} \frac{\log Tr\left(A^n(X)\right)}{n}$$

and $h_{top}(X) = 0$ if $X = \emptyset$, where $A(X)$ is the transition matrix of $X$, [8]. We remark that the transition matrix $A(X)$ describes the dynamics between the nodes of the network, which is represented by a graph $G$. The Perron-Frobenius Theorem states that, if the adjacency matrix $A \neq 0$ is irreducible and $\lambda_A$ is the Perron value of $A$, then $h_{top}(X) = \log \lambda_A$. We calculate the topological entropy of the associated dynamical system, which is equal to the logarithm of the growth rate of the number of admissible words, [8]. If we have a network associated to a graph $G$, which determine the shift space $X$, we will call network topological entropy of $G$ the quantity $h_{top}(X)$, i.e.,

$$h_{top}(G) = h_{top}(X) = \log \lambda_A. \tag{3}$$

The following result establishes a topological order in networks.

**Proposition 1.** *Let $G_1$ and $G_2$ be two undirected graphs, with the same number of vertices $N$, and $G_1$ be a not complete graph. If the graph $G_2$ is obtained from the graph $G_1$ by joining an edge, then $h_{top}(G_2) > h_{top}(G_1)$.*

*Proof.* Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be the adjacency matrices of the graphs $G_1$ and $G_2$, respectively. If the graph $G_2$ is obtained from the graph $G_1$ by joining an edge, then the adjacency matrix $B$ is obtained from the adjacency matrix $A$ by replacing some entry $a_{ij} = 0$ by $b_{ij} = 1$. As the graphs $G_1$ and $G_2$ are not directed, then the matrices $A$ and $B$ are symmetric, and $b_{ji} = 1$. Thus, the matrix $B$ is equal to the matrix $A$ plus some matrix with non negative entries. For any power $n$, we have $B^n = A^n + C$, for some matrix $C$ whose entries are all non negative. As $Tr(C) \geq 0$ and $Tr(B^n) = Tr(A^n) + Tr(C)$, then $Tr(B^n) > Tr(A^n)$. Consequently, we obtain $\frac{\log\ Tr(B^n)}{n} > \frac{\log\ Tr(A^n)}{n}$, for all integers $n$. From the definition of network topological entropy, Eq.(3), we have $h_{top}(G_2) > h_{top}(G_1)$.

## 4 Numerical simulations

In this section we will consider, as an example, a network with $N = 6$ nodes, having in each node the same skew-tent map, $f : [0,1] \to [0,1]$, defined by

$$f(x) = \begin{cases} \frac{x}{a} & \text{, if } 0 \leq x \leq a \\ \frac{1-x}{1-a} & \text{, if } a < x \leq 1 \end{cases}, \tag{4}$$

with $0.5 \leq a < 1$, see [6]. See Fig.1 where we present some examples of these networks. We start with a network of 7 edges and without the edges $e_{12}$, $e_{35}$, $e_{56}$, $e_{34}$, $e_{46}$, $e_{25}$, $e_{36}$, $e_{24}$ and each time we add the last edge of this list, we evaluate the eigenvalues of the Jacobian matrix, the Lyapunov exponents, the region where all transversal Lyapunov exponents are negatives, the synchronization interval and the mutual information rate for all communication channels of these networks. In order to compare the results, as we add one edge, we consider for all studied cases the same value $a = 0.9$ of the skew-tent map parameter. For this network, the region where all transversal Lyapunov exponents are negatives do not intersect the line $a = 0.9$. So, for this value of $a$ there is no synchronization interval, see 1) of Fig.2 and we do not evaluate the mutual information rate in this case.

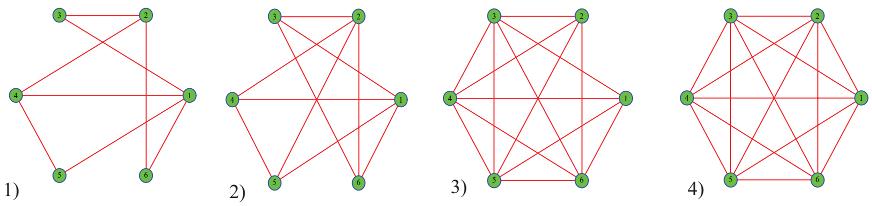


**Fig. 1.** Construction of networks by successively joining one edge, with 8, 10, 14 and 15 edges and $N = 6$ nodes.

We present the details for the network with 8 edges shown in 1) of Fig.1. The adjacency matrix $A$ and the Laplacian matrix $L$ of this network are:

$$A = \begin{bmatrix} 0\,0\,1\,1\,1\,1 \\ 0\,0\,1\,1\,0\,1 \\ 1\,1\,0\,0\,0\,0 \\ 1\,1\,0\,0\,1\,0 \\ 1\,0\,0\,1\,0\,0 \\ 1\,1\,0\,0\,0\,0 \end{bmatrix} \quad \text{and} \quad L = A - D = \begin{bmatrix} -4 & 0 & 1 & 1 & 1 & 1 \\ 0 & -3 & 1 & 1 & 0 & 1 \\ 1 & 1 & -2 & 0 & 0 & 0 \\ 1 & 1 & 0 & -3 & 1 & 0 \\ 1 & 0 & 0 & 1 & -2 & 0 \\ 1 & 1 & 0 & 0 & 0 & -2 \end{bmatrix},$$

where $D$ is the diagonal matrix with entries $d_{ii} = k_i$, beeing $k_i$ the degree of each node $i$. This network is defined by the following system,

$$\begin{cases} \dot{x}_1 = f(x_1) + \sigma(-4x_1 + x_3 + x_4 + x_5 + x_6) \\ \dot{x}_2 = f(x_2) + \sigma(-3x_2 + x_3 + x_4 + x_6) \\ \dot{x}_3 = f(x_3) + \sigma(x_1 + x_2 - 2x_3) \\ \dot{x}_4 = f(x_4) + \sigma(x_1 + x_2 - 3x_4 + x_5) \\ \dot{x}_5 = f(x_5) + \sigma(x_1 + x_4 - 2x_5) \\ \dot{x}_6 = f(x_6) + \sigma(x_1 + x_2 - 2x_6) \end{cases},$$

where $\sigma$ is the coupling parameter. The Jacobian matrix is given by,

$$J = \begin{bmatrix} c-4\sigma & 0 & \sigma & \sigma & \sigma & \sigma \\ 0 & c-3\sigma & \sigma & \sigma & 0 & \sigma \\ \sigma & \sigma & c-2\sigma & 0 & 0 & 0 \\ \sigma & \sigma & 0 & c-3\sigma & \sigma & 0 \\ \sigma & 0 & 0 & \sigma & c-2\sigma & 0 \\ \sigma & \sigma & 0 & 0 & 0 & c-2\sigma \end{bmatrix},$$

being $c = c(x)$ the slope of $f$, Eq.(4), given by $c(x) = \frac{1}{a}$, if $x \leq a$ and $c(x) = -\frac{1}{1-a}$ if $x > a$. The eigenvalues of the Jacobian are $\mu_1 = c$, $\mu_2 = c - 4\sigma$, $\mu_3 = c - 3\sigma$, $\mu_4 = c - 2\sigma$, $\mu_5 = c - \frac{7}{2}\sigma - \frac{\sqrt{17}}{2}$ and $\mu_6 = c - \frac{7}{2}\sigma - \frac{\sqrt{17}}{2}$. The first eigenvector is $(1, 1, 1, 1, 1, 1)$ and it corresponds to the parallel Lyapunov exponent $\lambda_{\|}$. The others eigenvectors correspond to the transversal Lyapunov exponents $\lambda_{\perp_i}$, with $i = 2, 3, 4, 5, 6$. So, the parallel Lyapunov exponent is

$$\lambda_{\|} = \int \ln|\mu_1| = \int_0^a \ln\frac{1}{a} + \int_a^1 \ln\left|\frac{-1}{1-a}\right| = -a\ln a - (1-a)\ln(1-a) \qquad (5)$$

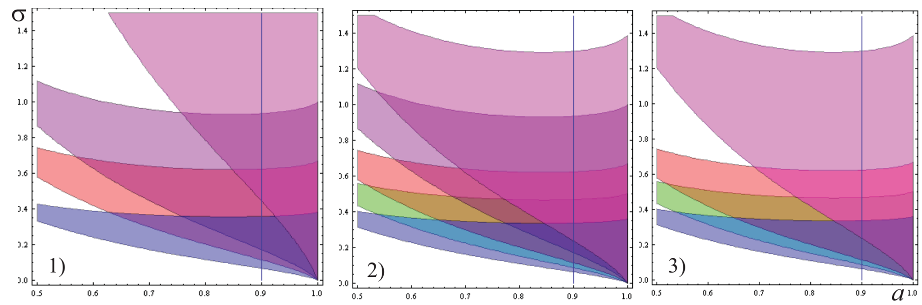


**Fig. 2.** Regions where the transversal Lyapunov exponents are negatives. The synchronization region is the intersection of these regions. In the vertical axis is the coupling parameter $\sigma$ and in the horizontal axis is the tent map parameter $a$. In 1) is the network with 7 edges, in 2) with 8 edges, and in 3) with 9 edges. The image in 1) shows that for $a = 0.9$ there is no synchronization interval because the intersection of the regions where all transversal Lyapunov exponents are negatives does not occur for $a = 0.9$.

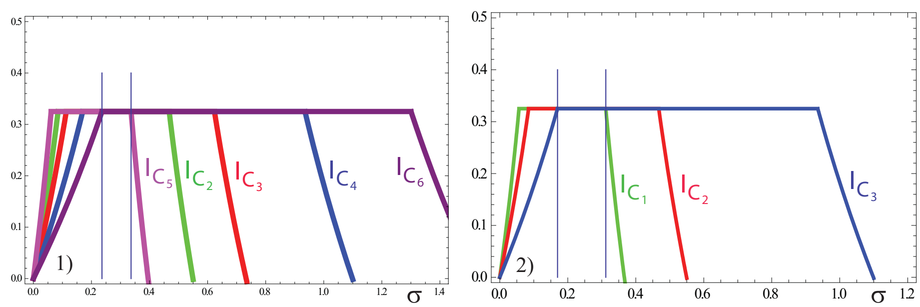


**Fig. 3.** $I_{C_i}$ for the network with 8 edges in 1) of Fig.1 and with 10 edges in 2) of Fig.1.

and the transversal Lyapunov exponents are

$$\lambda_{\perp_i} = a \ln \left| \frac{1}{a} - \nu_i \sigma \right| + (1-a) \ln \left| -\frac{1}{1-a} - \nu_i \sigma \right|, \quad \text{with} \quad i = 2, 3, 4, 5, 6$$

where $\nu_2 = 4$, $\nu_3 = 3$, $\nu_4 = 2$, $\nu_5 = \frac{7}{2}\sigma + \frac{\sqrt{17}}{2}$ and $\nu_6 = \frac{7}{2}\sigma - \frac{\sqrt{17}}{2}$. In order to have synchronization, all transversal Lyapunov exponents must be negatives, see 2) in Fig.2. In this figure, each color corresponds to a region where one of the transversal Lyapunov exponents is negative. For example, if $a = 0.9$, then the synchronization interval is $]0.236, 0.336[$, where all the transversal Lyapunov exponents $\lambda_{\perp_i}$ are negative. See also 3) in Fig.2 for the network with 9 edges. To evaluate the mutual information rate (MIR), according to Eq.(2), for each $\lambda_{\perp_i}$ we obtain the interval $]a_i, b_i[$ where $\lambda_{\perp_i} < 0$, thus

$$I_{C_i} = \begin{cases} -a \ln a - (1-a)\ln(1-a) - a \ln \left| \frac{1}{a} - \nu_i \sigma \right| - (1-a) \ln \left| -\frac{1}{1-a} - \nu_i \sigma \right|, \\ \qquad\qquad\qquad\qquad \text{if} \ \ \sigma < a_i \ \ \text{or} \ \ \sigma > b_i \\ -a \ln a - (1-a)\ln(1-a), \ \ \text{if} \ \ a_i < \sigma < b_i \end{cases}$$

with $a = 0.9$ and $i = 2, 3, 4, 5, 6$. See in 1) of Fig.3 the plots of these $I_{C_i}$. The MIR attains its maximum 0.325..., in an interval of lenght 1.028 and the network topological entropy, given by Eq.(3), is $\log \lambda_A = 1.02835...$.

The second case that we study in detail is the network with 10 edges and without the edges $e_{12}$, $e_{35}$, $e_{56}$, $e_{34}$, $e_{46}$, see 2) of Fig.1. The adjacency matrix $A$ and the Laplacian matrix $L$ are given by,

$$A = \begin{bmatrix} 0\,0\,1\,1\,1\,1 \\ 0\,0\,1\,1\,1\,1 \\ 1\,1\,0\,0\,0\,1 \\ 1\,1\,0\,0\,1\,0 \\ 1\,1\,0\,1\,0\,0 \\ 1\,1\,1\,0\,0\,0 \end{bmatrix} \quad \text{and} \quad L = A - D = \begin{bmatrix} -4 & 0 & 1 & 1 & 1 & 1 \\ 0 & -4 & 1 & 1 & 1 & 1 \\ 1 & 1 & -3 & 0 & 0 & 1 \\ 1 & 1 & 0 & -3 & 1 & 0 \\ 1 & 1 & 0 & 1 & -3 & 0 \\ 1 & 1 & 1 & 0 & 0 & -3 \end{bmatrix}.$$

This network is defined by the system,

$$\begin{cases} \dot{x}_1 = f(x_1) + \sigma(-4x_1 + x_3 + x_4 + x_5 + x_6) \\ \dot{x}_2 = f(x_2) + \sigma(-4x_2 + x_3 + x_4 + +x_5 + x_6) \\ \dot{x}_3 = f(x_3) + \sigma(x_1 + x_2 - 3x_3 + x_4) \\ \dot{x}_4 = f(x_4) + \sigma(x_1 + x_2 - 3x_4 + x_5) \\ \dot{x}_5 = f(x_5) + \sigma(x_1 + x_2 + x_4 - 3x_5) \\ \dot{x}_6 = f(x_6) + \sigma(x_1 + x_2 + x_3 - 3x_6) \end{cases},$$

and the Jacobian matrix is given by

$$J = \begin{bmatrix} c-4\sigma & 0 & \sigma & \sigma & \sigma & \sigma \\ 0 & c-4\sigma & \sigma & \sigma & \sigma & \sigma \\ \sigma & \sigma & c-3\sigma & \sigma & 0 & 0 \\ \sigma & \sigma & \sigma & c-3\sigma & \sigma & 0 \\ \sigma & \sigma & 0 & \sigma & c-3\sigma & 0 \\ \sigma & \sigma & \sigma & 0 & 0 & c-3\sigma \end{bmatrix}.$$

The eigenvalues of the Jacobian matrix are $\mu_1 = c$, $\mu_2 = c - 6\sigma$, $\mu_3 = \mu_4 = \mu_5 = c - 4\sigma$ and $\mu_5 = c - 2\sigma$. Thus, the parallel Lyapunov exponent is identical to the previous case, Eq.(5), and the transversal Lyapunov exponents are

$$\lambda_{\perp_i} = a \ln\left|\frac{1}{a} - \nu_i\sigma\right| + (1-a)\ln\left|-\frac{1}{1-a} - \nu_i\sigma\right|, \quad \text{with} \quad i = 2, 3, 4$$

where $\nu_2 = 6$, $\nu_3 = 4$ and $\nu_4 = 2$. See 1) in Fig.4 the regions where these transversal Lyapunov exponents are negatives. For $a = 0.9$, this network synchronizes if $\sigma \in ]0.170, 0.312[$. We compute the $I_{C_i}$ like in the previous case and we plot its graphics in 2) of Fig.3. The MIR attains its maximum 0.325..., in an interval of lenght 1.216 and the network topological entropy is $\log \lambda_A = 1.21559....$ Figs.4, 5 and Table 1 contain information similar to the other cases analyzed in this topological order.

## 5 Conclusions and discussion

We started our simulations, considering the network with 8 edges and without the edges $e_{12}$, $e_{35}$, $e_{56}$, $e_{34}$, $e_{46}$, $e_{25}$, $e_{36}$ and in each step we add the last edge

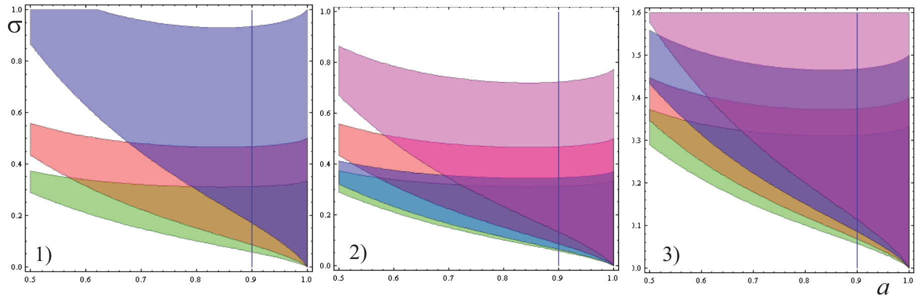


**Fig. 4.** Regions where the transversal Lyapunov exponents are negatives. The synchronization region is the intersection of these regions. In 1) is the network with 10 edges, in 2) with 11 edges, and in 3) with 12 edges. For the same value of $a$, the amplitude of the synchronization interval increases.

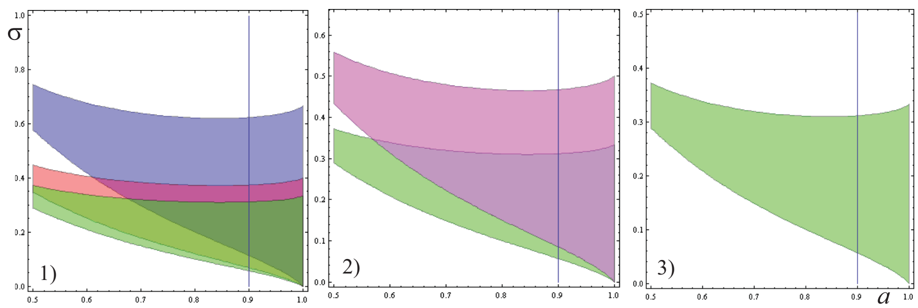


**Fig. 5.** Regions where the transversal Lyapunov exponents are negatives. The synchronization region is the intersection of these regions. In 1) is the network with 13 edges, in 2) with 14 edges and in 3) with 15 edges (complete network). For the same value of $a$, the amplitude of the synchronization interval increases.

| Edges | $\mu_i = c - \nu_i \sigma \ (i = 2, 3, 4, 5, 6)$ | Sync. interv. | Ampl. | $\log \lambda_A$ |
|---|---|---|---|---|
| 8 | $\nu_2 = 4,\ \nu_3 = 3,\ \nu_4 = 2,\ \nu_5 = \frac{7+\sqrt{17}}{2},\ \nu_6 = \frac{7-\sqrt{17}}{2}$ | ]0.236,0.336[ | 0.100 | 1.028 |
| 9 | $\nu_2 = \nu_3 = 4,\ \nu_4 = 3,\ \nu_5 = \frac{7+\sqrt{17}}{2},\ \nu_6 = \frac{7-\sqrt{17}}{2}$ | ]0.236,0.336[ | 0.100 | 1.127 |
| 10 | $\nu_2 = 6,\ \nu_3 = \nu_4 = \nu_5 = 4,\ \nu_6 = 2$ | ]0.170,0.312[ | 0.142 | 1.216 |
| 11 | $\nu_2 = 6,\ \nu_3 = \nu_4 = 4,\ \nu_5 = 4 + \sqrt{2},\ \nu_6 = 4 - \sqrt{2}$ | ]0.131,0.312[ | 0.181 | 1.312 |
| 12 | $\nu_2 = \nu_3 = 6,\ \nu_4 = 5,\ \nu_5 = 4,\ \nu_6 = 3$ | ]0.113,0.312[ | 0.199 | 1.403 |
| 13 | $\nu_2 = \nu_3 = \nu_4 = 6\ \nu_5 = \nu_6 = 4$ | ]0.085,0.312[ | 0.227 | 1.475 |
| 14 | $\nu_2 = \nu_3 = \nu_4 = \nu_5 = 6\ \nu_6 = 4$ | ]0.085,0.312[ | 0.227 | 1.548 |
| 15 | $\nu_2 = \nu_3 = \nu_4 = \nu_5 = \nu_6 = 6$ | ]0.057,0.312[ | 0.255 | 1.609 |

**Table 1.** Jacobian eigenvalues, $\mu_i$, for $(i = 2, 3, 4, 5, 6)$, which correspond to the transversal Lyapunov exponents, synchronization interval, its amplitude, network topological entropy and the number of edges from 8 to 15 (complete network).

of this list. In each step of this construction, we obtain the Laplacian matrix and compute the eigenvalues $\mu_i$ $(i = 1, 2, 3, 4, 5, 6)$ of the Jacobian matrix, the parallel and transversal Lyapunov exponents, the synchronization interval, the network topological entropy and the $I_{C_i}$ for the networks with 8, 9, 10, 11, 12, 13, 14 and 15 edges (complete network). For all these cases $\mu_1 = c$

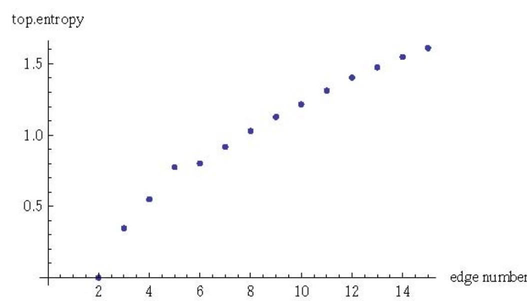


**Fig. 6.** The network topological entropy increases as the the number of edges of the network increases.

and this eigenvalue correspond to the synchronization manifold. The others $\mu_i$ correspond to the transversal Lyapunov exponents and are presented in Table 1. In this table is also presented the synchronization interval and the network topological entropy, for all these cases. See in Figs.2, 4 and 5 the synchronization regions, in terms of the skew-tent map parameter $a$ and of the coupling parameter $\sigma$. In Fig.6 we may see that the network topological entropy increases as we add one edge sucessively to the network, which confirms Proposition 1.

From the numerical simulations shown in figures and Table 1, we conclude that, with the topological order established, the interval where the mutual information rate attains its maximum, the synchronization interval, increases its amplitude. Thus, we claim that:

**Conjecture:** As larger the network topological entropy, the larger is the rate with which information is exchanged between nodes in the network.

## Acknowledgment

This work is funded by FCT through the National Funds - Foundation for Science and Technology within the project PEst-OE/MAT/UI0006/2011, CIMA-UE, CEAUL and ISEL.

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