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Chaotic Dissipative Raman Solitons

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Abstract. A new type of dissipative solitons is announced. The numerical simulations based on the generalized nonlinear complex Ginzburg-Landau equation demonstrate that this soliton (dissipative Raman soliton, DRS) develops in the normal dispersion regime under effect of the stimulated Raman scattering (SRS). The SRS causes the red-shift and re-shaping of a soliton spectrum as well as changes a soliton group velocity. The main effects of SRS on a dissipative soliton are the chaotization of a soliton dynamics, the automodulational fragmentation of a soliton envelope and the enhancement of a tendency to multiple pulsing. As a result, a DRS becomes "noisy" and loses a property of energy scalability that troubles a high-energy pulse generation from mode-locked ber lasers.
Keywords: dissipative solitons, stimulated Raman scattering, chaotic modeling, generalized nonlinear complex Ginzburg-Landau equation.

1 Introduction

In the last decade, the concept of a dissipative soliton (DS), that is a strongly localized and stable structure emergent in a nonlinear dissipative system far from the thermodynamic equilibrium was actively developing and became well-established. This concept is highly useful in very different fields of science ranging from field theory and cosmology, optics and condensed-matter physics to biology and medicine [1]. Non-equilibrium character of a system, where the DSs emerge requires from them a well-organized energy exchange with an environment. In an optical system, the resonant and nonlinear coupling with such an environment causes a number of effects, in particular, the stimulated Raman scattering (SRS). In the last case, a light (photons) propagating through some medium (e.g., fiber) is scattered by oscillatory modes (phonons) of the latter [2]. As was found, the SRS can affect the DSs dramatically [3].

In this work, the results of numerical analysis of the DS dynamics affected by a strong SRS are presented. The testbed for such an analysis is the generalized nonlinear complex Ginzburg-Landau equation (generalized NCGLE), which is a common NCGLE (e.g., see [4]) supplemented with the SRS term in a general form as well as with the term describing a white quantum noise. To the best of

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our knowledge, such a stochastic generalized NCGLE is considered for the first time.

The analysis demonstrates that the DS stability changes drastically, when the SRS becomes strong that is when the DS energy is large. The SRS destabilizes a DS and causes chaotization of its dynamics. The most interesting result is that a new type of DS develops in the presence of SRS. This DS can be named "dissipative Raman soliton" (DRS) because it is i) frequency down-shifted and ii) has a strongly inhomogeneous phase (i.e., "chirped"). The last property indicates the strong coupling of a DRS with an environment (i.e., this soliton is dissipative indeed). Also, it is found that the DRS dynamics is chaotic.

2 Generalized nonlinear complex Ginzburg-Landau equation

Simplest and most studied models for nonequilibrium phenomena in nonlinear systems are based on the different versions of NCGLE [1,5,6]. We will consider the following generalized version of the cubic-quintic NCGLE:

$$\begin{aligned} &\tfrac{\partial a(z,t)}{\partial z} = i\left[\tfrac{\beta}{2}\tfrac{\partial^2}{\partial t^2} - (1-f_R)\,\gamma|a\,(z,t)|^2\right] a\,(z,t) + \\ &+\left[-\sigma + \alpha\tfrac{\partial^2}{\partial t^2} + \kappa\left(1-\zeta|a\,(z,t)|^2\right)|a\,(z,t)|^2\right] a\,(z,t) - \\ &-i f_R \gamma a\,(z,t) \int\limits_{-\infty}^{\infty} dt' R\,(t')\,|a\,(z,t-t')|^2 + \chi\,(z,t)\,. \end{aligned} \qquad (1)$$

In particular, Eq. (1) can be interpreted in the following way. $a(z,t)$ is a slowly varying amplitude of light wave package, where z and t are a longitudinal propagation distance and a "local time", respectively. In a laser, the propagation distance is simply resonator round-trip number in the framework of the distributed model. The local time is associated with a group velocity of wave package (e.g., see [7]). First row of Eq. (1) is a so-called nonlinear Schrödinger equation and describes the nondissipative factors such as a group velocity dispersion with the coefficient β and a self-phase modulation with the coefficient γ. Second row generalizes the nonlinear Schrödinger equation with taking into account the dissipative factors such as a saturable loss with the coefficient σ, a spectral dissipation with the coefficient α and a nonlinear gain with the coefficient κ. The nonlinear gain is saturable (the coefficient of saturation is ζ). The saturable net-loss is supposed to be energy-dependent: $\sigma = \varepsilon\left(\int\limits_{-\infty}^{\infty} dt'\,|a\,(z,t')| \Big/ E_s - 1\right)$, where $\varepsilon = 0.05$ and E_s is a variable parameter defining the energy inflow in a system. These two rows of Eq. (1) give the cubic-quintic NCGLE, which is the basic model for analysis of an ultrashort pulse generation in both solid-state and fiber lasers.

We invent two physically relevant sophistication of the common cubic-quintic NCGLE (third row in Eq. (1)). 1) Since both amplification and dissipation in a laser produce inevitably the quantum fluctuations, Eq. (1) has to be stochastic that is provided by inclusion of the stochastic term χ. This term describes a complex white noise with the correlation function

$\langle \chi (z', t') \chi^* (z, t) \rangle = \mathit{\Gamma} \delta (z' - z) \delta (t' - t)$ (the noise "power" is $\mathit{\Gamma} = 10^{-10}/\gamma$ in our case). 2) At the high energy levels, the SRS becomes strong in fiber lasers [3]. Its influence is taken into account by the first term in third row of Eq. (1). We do not use the common approximation for this term in the form of the Taylor series expansion (e.g., see [8]). That allows the adequate description of an frequency conversion and energy flows for a DS affected by SRS.

In Eq. (1), the SRS is characterized by the response function [2]

$$R(t) = \frac{T_1^2 + T_2^2}{T_1 T_2^2} e^{-t/T_2} \sin\left(\frac{t}{T_1}\right), \tag{2}$$

where $T_1 = 12.2$ fs defines the Stokes frequency and $T_2 = 32$ fs defines the width of a Stokes line. The parameter $f_R = 0.22$ is defined by the Raman gain. All these numerical values correspond to a fused silica.

Eq. (1) was solved numerically by the symmetrized Fourier split-step method. The integral in Eq. (1) was evaluated in the Fourier domain on the basis of the convolution theorem. The size of temporal window and the propagation step were varied, the local time step was equal to 1 fs.

3 Results and discussion

The Raman lines in a fiber form a broad joined line that corresponds to a comparatively large T_2. Since the DS developing in the normal dispersion regime (β >0 in Eq. (1)) is stretched due to a large chirp, its width $T \gg T_2$ and one may expect that the SRS will play a substantial role in the dynamics of such solitons. The reason is that the group velocities of a Raman pulse and an ordinary DS differ due to dispersion. As a result, a Raman pulse must have an ample time for amplification during a time period, when it and a DS are overlapping [9]. On the other hand, the Raman frequency shift in a fiber is comparatively large (small T_1). But the DS spectral width is large as well (again due to a large chirp). Therefore one may expect an effective interpulse Raman scattering [2] in this case.

The calculations demonstrate that the SRS begins to contribute nontrivially into the DS dynamics and properties, when the DS energy $E \equiv \int_{-\infty}^{\infty} |a(z, t')|^2 dt'$ exceeds some threshold value (≈20 nJ in an all-fiber laser [9]). In the model under consideration, the DS energy is defined mainly by the parameter E_s. As was demonstrated in Ref. [10], the DS parametric space of the cubic-quintic NCGLE is two-dimensional and the relevant coordinates of this space are: $E' \equiv E_s \kappa^{3/2} \sqrt{\zeta}/\gamma\sqrt{\alpha}$ and $C \equiv 2\alpha\gamma/\beta\kappa$. Below, this dimensionless representation of the DS parametric space will be used.

For some fixed energy E_s, the multiple DSs appear when the dispersion β is relatively small (Fig. 1). These DSs redistribute an overall energy so that the energy of individual soliton becomes relatively small. As a result, the SRS does not affect their dynamics: there are no an extra group-delay and a transformation of spectrum.

The dispersion growth suppresses multipulsing so that a sole DS develops. Further growth of dispersion increases the difference of velocities between the

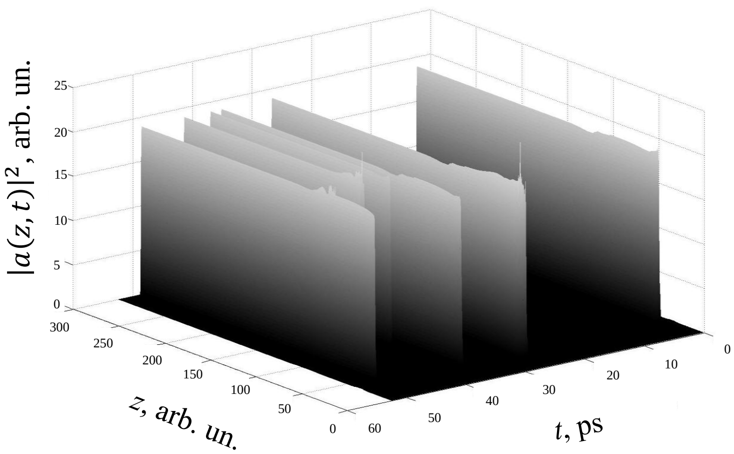

Fig. 1. Evolution of multiple DSs for E' =111 and C =0.732.

red and blue spectral components of a chirped pulse (remind that the DS phase is inhomogeneous) that stretches a DS. If the energy is sufficiently large, such a pulse becomes flat-top that corresponds to the energy scalable regime, when the peak power is fixed ($\approx 1/\zeta$). An energy scalable DS with the fixed peak power can remain stable if the energy growth, provided by energy inflow from an environment is compensated by the DS broadening [11].

For a comparatively small dispersions, i.e. in the vicinity of multipulsing threshold, the SRS causes i) pulse acceleration (i.e., growth of the group velocity in comparison with that of ordinary DS), ii) irregular perturbations at pulse traveling edge (where the blue spectral components are located), iii) chaotical evolution of the DS peak power, and iv) DS spectrum splitting (Fig. 2).

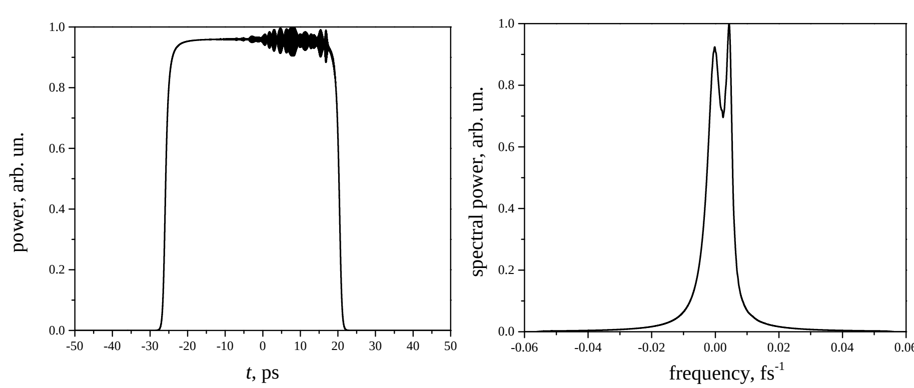

Fig. 2. Left: DS profile $|a(z = const, t)|^2$. Right: DS spectral power vs. frequency deviation from the central frequency corresponding to zero spectral dissipation (see second term of second row in Eq. (1)). E' =111 and C =0.59. Physically, the parameters correspond to an Yb-all-fiber laser with a 40 nm gain bandwidth.

Further dispersion growth enhances the DS acceleration. The pulse becomes noisy that is its envelope is strongly and irregularly perturbed and resembles a glass of boiling water (Fig. 3, left). As a consequence, the peak power evolves chaotically. Simultaneously, the regular ("solitonic") part of the spectrum shifts into red-domain while the blue spike becomes intensive and irregular (Fig. 3, right).

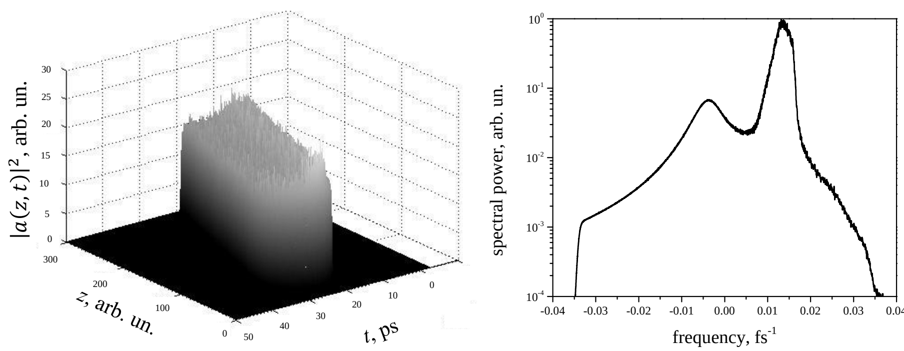

Fig. 3. Left: evolution of the DS profile $|a(z,t)|^2$. Right: logarithm of the averaged DS spectral power vs. frequency deviation. Averaging is performed over the interval Δz =2000 with the step δz =10. E' =111 and C =0.244.

Unlike the regime without SRS, larger dispersions cause the multipulsing yet again (Fig. 4, left) so that the domain of single pulse generation becomes confined along $C-$dimension. The chaotization of the peak power evolution and the perturbations of the DS traveling edge increase in parallel with the dispersion growth. The red part of the spectrum rises and shifts to lower frequencies.

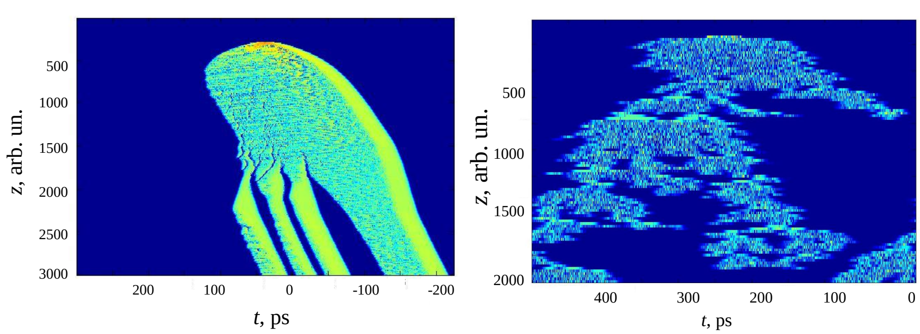

Fig. 4. Contour plot of the DS power evolution with (left) and without (right) SRS. Left: E' =111, C =0.22 and F =0.22. Right: E' =111, C =0.0366 and F =0.

Further increase of dispersion leads to a generation of DRS complexes. The example of such complex is shown in Fig. 5. Again, the traveling DRS edge is perturbed that causes chaotic changes in the peak power evolution (Fig. 6, left). The chaotic character of evolution can be identified by the continuum-like RF spectrum of peak power set (Fig. 6, right). The spectrum splits in two separated parts shifted in opposite sides relatively the central frequency. Both parts have truncated edges (see inset in Fig. 5, right) that is the typical property of a chirped DS. The main part of the spectrum is red-shifted and modulated due to interference between DRSs.

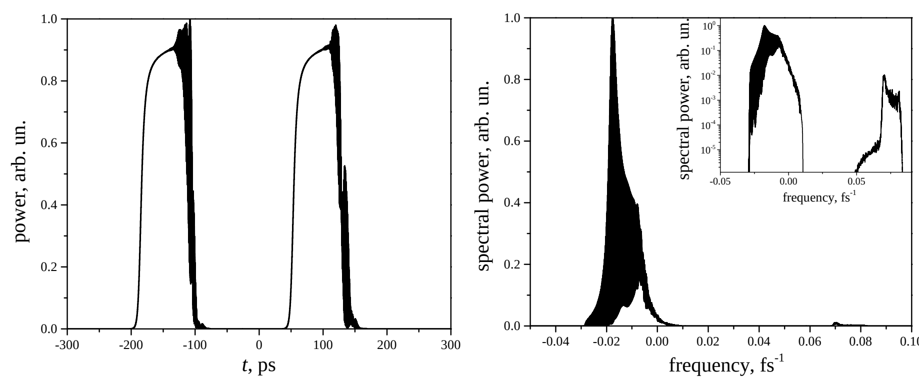

Fig. 5. Left: DS profile $|a(z = const, t)|^2$. Right: averaged DS spectral power vs. frequency deviation. Averaging is performed over the interval Δz =1000 with the step δz =10. Inset shows the spectrum on a logarithmic scale. E' =111 and C =0.0366.

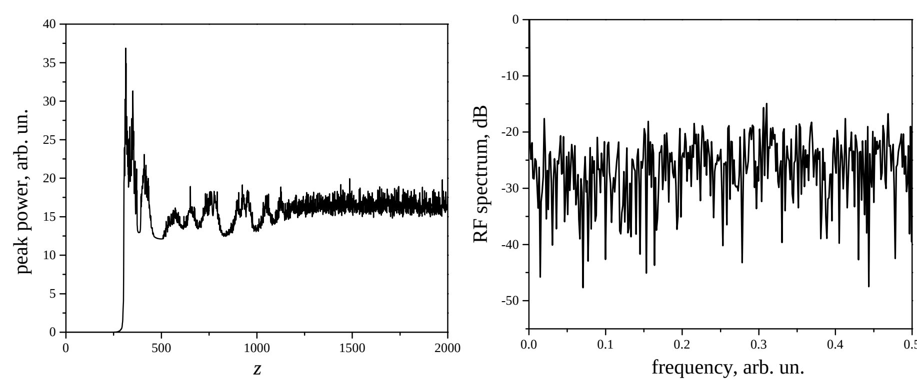

Fig. 6. Left: Evolution of DRS peak power. Right: RF spectrum of the peak power set over last interval Δz =750. E' =111 and C =0.0366.

Why do we name the entity in Fig. 5 as a "dissipative Raman soliton"? First of all, one has to note that the spectrum in Fig. 5, right is a sum of almost identical spectra of individual pulses in Fig. 5, left. That is each pulse has a spectrum with both Stokes- and anti-Stokes "steep hills". Such a spectrum is not possible in an ordinary dissipative system with only self-phase modulation, second-order dispersion and spectral dissipation terms. Moreover, the main part of energy is concentrated in the lower frequency domain. All these observations suggest that the SRS contributes substantially into the pulse properties, both temporal and spectral. At last, truncated shape of the spectrum (inset in Fig. 5, right) suggests that the pulse is strongly chirped, i.e. it has a substantially inhomogeneous phase [10]. But the last fact testifies the nontrivial energy flows inside a pulse as well as between a pulse and environment [4]. Also, one has to add that there exists no DS without SRS for the parametrical set of Figs. 5, 6. The dynamics becomes completely chaotic without SRS in this case (Fig. 4, right). It is clear, that the SRS plays a crucial role in the DS stabilization for the large dispersions due to some "negative passive feedback" provided by spectral shift from the minimum of the frequency dissipation. Thus, one may conclude that the SRS is a formative factor for the DS considered so that such a soliton can be named a "dissipative Raman soliton".

A passive negative feedback produced by the combined action of SRS and spectral dissipation enhances the tendency to multipulsing. The energy of individual DS in the multiple pulse complex is lower than that of single DS. Therefore, the frequency shift due to SRS is lower, as well. As a result, the spectral loss is lower too. Thus, the multipulsing becomes more advantageous energetically. This tendency to multipulsing in combination with the chaotization of DS dynamics in the presence of SRS confine the DS energy scalability.

4 Conclusion

For the first time to our knowledge, a new type of dissipative solitons of the generalized cubic-quintic nonlinear complex Ginzburg-Landau equation was described and analyzed. Such solitons emerge under action of stimulated Raman scattering in the presence of white quantum noise and can be named "dissipative Raman solitons". Changes in the DRS characteristics with the dispersion growth were traced and a complicated structure of the region, where DRS exists, was established. It was found that the dynamics of DRS is chaotic due to irregular perturbation at the soliton traveling edge. A two-compound character of the DRS spectrum was revealed so that the soliton spectrum consists of Stokes- and anti-Stokes spices with truncated edges. The last fact suggests that the DRS considered is strongly chirped. It was demonstrated that the DRS can exist in the regions of large dispersion where an ordinary DS does not emerge. Simultaneously, the SRS leads to an additional spectral loss in a system with the spectral dissipation. This confines a DS energy scalability.

Acknowledgments

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