Influence of Frequency of Excitation on Bifurcational Behaviour of an Experimental 2-DoF Mechanical System with Stick-Slip Friction

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Abstract: Influence of frequency of excitation on chaotic dynamics of a 2-DoF mechanical system with dry friction is investigated. The analysed system consists of a block vibrating on a transmission belt driven by an electric motor. Stick-slip friction in a contact between the block and the belt introduce significant variability of load affecting operation of the driving system. In the naturally coupled systems, a resultant unsteady rotational velocity of the DC motor acts as a time varying high frequency disturbance of velocity of motion of the base. Bifurcational behaviour of the 2-DoF block-on-belt system with dry friction and its response to an irregular excitation caused by the disturbed velocity of motion of the base is analysed using bifurcation diagrams. Mathematical model of the block-on-belt system with a normal force intensification mechanism and the electric motor based driving system has been developed and numerically solved. Changes in the assumed stiffness parameter of bifurcation of the investigated stick-slip mechanical system with dry friction have provided some comparable bifurcation diagrams bringing interesting observations and conclusions.
Keywords: Stick-slip dynamics, Bifurcational analysis, Numerical simulations, Experimental measurements

1 Introduction

Nonlinearities with their origin in backlashes, impacts, stick-slip motions, or interactions with a discrete control systems are present in physical systems.

A lot of scientific work has been done in the frame of analysis of complex behaviour exhibited by vibrating mechanical systems with nonlinearities [1-8,10-15,17-19]. Methods of dynamical analysis have found their place not only in mechanics, but also in electronics [16] and even in biology [9].

This paper takes a research on the influence of frequency of excitation on the possibility of appearance of chaotic dynamics in a 2-DoF mechanical system with friction. Time-varying frequency of excitation of moving base in the block-on-belt model, providing the disturbed linear velocity of motion of the base, comes from the modelled real behaviour of the driving system installed on a

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laboratory stand being constructed for investigation of frictional effects [1-3]. It makes our simulations more realistic by proving visible changes in the investigated system dynamics. Two mathematical models are taken into account in this work, i.e.: 1) a model without coupling between the mass oscillating on the moving base and the system driving the base; 2) a model with coupling, which states a kind of feedback dynamics. Results of our dynamical analysis of both systems are compared on bifurcation diagrams.

2 Modelling of the Investigated Mechatronic System

The investigated mechatronic system is depicted in Fig.1. It consists of three interconnected subsystems, such as: 1) an electric motor; 2) the transmission system consisting of a worm gear, conveyer belt and a pulley; 3) the body (a block) oscillating on the conveyer belt with a friction force intensification mechanism realised by single pendulum (a bracket rotating about the pivot *S*). The motor is connected to a worm gear, which by a toothed belt transmits the driving torque to the belt pulley. A block of mass *m* is connected to a fixed wall by the linear spring of stiffness k_1 and to the pendulum body of mass *M* by means of springs of the stiffness k_2 and k_3. The virtual dashpots of the damping c_1 and c_2 model unknown effects of the system's viscous damping in the bearings and resistance of motion in the air.

The linear velocity V_b of the conveyer belt (the moving base) plays the role of the time-varying excitation of the oscillating block, and hence, self-sustained vibrations of the block are observed. Detailed description of the system can be found in [1-3,6].

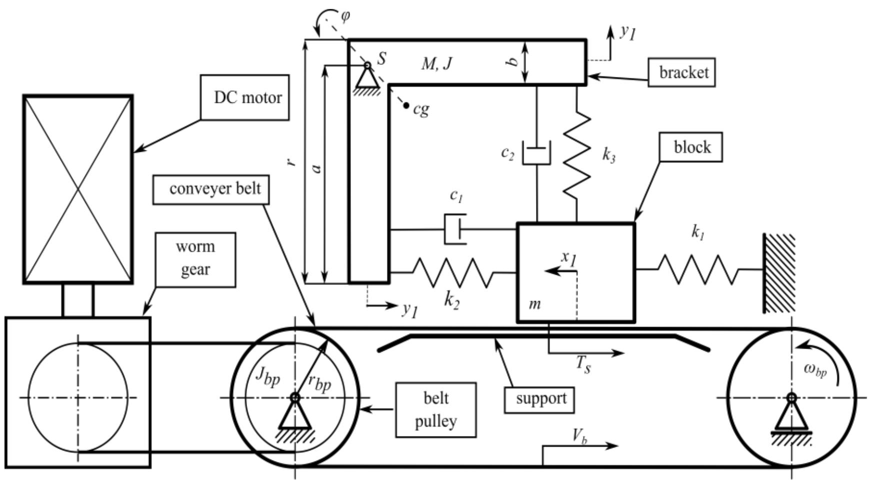


Fig. 1. Physical model of the investigated mechatronic system.

The electromechanical model of the electric DC motor follows:

$$\dot{i}_w L_w = U_{in} - k_e \omega(t) - i_w(t) R_w \tag{1}$$

$$\dot{\omega}(t) J_{dc} = k_m i_w(t) - M_l(t) - b\omega(t) - T_C(\omega(t)), \tag{2}$$

where: dot over symbols i and ω denotes the first derivative with respect to time, L_w – winding inductance, R_w – winding resistance, k_e – constant of electromotive force, k_m – motor torque constant, i_w – winding current, U_{in} – input voltage to the motor, ω – angular velocity of the rotor, J_{dc} – mass moment of inertia of the rotor, b – viscous friction coefficient, M_l – time varying torque loading rotor of the motor, T_C – Coulomb friction force given by the formula:

$$T_C(\omega) = \begin{cases} T_{st} & \text{for} \quad \omega(t) = 0, \\ -sign(\omega) T_k & \text{for} \quad \omega(t) \neq 0, \end{cases} \tag{3}$$

where: T_{st} – static friction force, T_k – kinetic friction force.

Equation of motion of the belt pulley is given in the form:

$$\dot{\omega}_{bp}(t) J_{bp} = M_2 - T_S(t) r_{bp}, \tag{4}$$

where: J_{bp} – mass moment of inertia of the pulley, ω_{bp} – pulley's angular velocity, r_{bp} – radius of the pulley, M_2 – output torque of worm gear, T_S –time-varying friction force caused by an irregular stick-slip motion of mass m.

Following equations constitute the simplified model of the worm gear:

$$M_2 = M_1 \eta \rho, \ \omega(t) = \omega_{bp}(t) \rho, \ \dot{\omega}(t) = \dot{\omega}_{bp}(t) \rho, \tag{5}$$

where: M_1 and M_2 – input and output torques of worm gear, ρ – a transmission ratio, η – efficiency of the angular velocity transmission system.

The total torque from the transmission system loading the motor follows:

$$M_l = M_1 = \frac{1}{\eta\rho} \left(\dot{\omega}(t) \frac{J_{bp}}{\rho} + T_S(t) r_{bp} \right). \tag{6}$$

Equation of motion of the transmission system reduced to the DC motor's rotor (shaft) is given:

$$\dot{\omega}(t) \left(J_{dc} + J_{bp} / (\eta\rho^2) \right) = k_m i_w(t) - r_{bp} / (\eta\rho) \cdot T_S(t) - b\omega(t) - T_C(\omega). \tag{7}$$

Equations (8)-(9) describe the dynamical behaviour of the body oscillating on the moving base and coupled with the normal force intensification mechanism created by single pendulum. The mechanism simulates behaviour of braking systems with intensification of friction force [1-3]. Derivation of equations of motion of the investigated physical system are provided in [11].

Two characteristic phases of movement in the examined system dynamics are distinguished. The "stick phase" of movement occurs when the block moves with constant velocity V_b – the linear velocity of motion of the conveyer belt. If the static friction force reaches its maximum value and it does no longer compensate for the resultant force generated by springs, then the stick contact between the mass and the moving base is lost, hence, the "slip phase" of the relative motion appears. Then, the block moves with an accelerated motion that is opposite directed to the motion of the base until the dynamic friction force will compensate the resultant force of springs and inertia of the sliding body.

The investigated two-degrees-of-freedom mechanical system with dry friction is described by the two second order differential equations:

$$\ddot{x}_1 m + \dot{z}_1 c_1 + (k_1 + k_2)x_1 + k_2 y_1 + \frac{k_3 x_1}{r}\left(y_1 + x_1^2/(2r)\right) = -T_S\,, \tag{8}$$

$$\ddot{y}_1 J/r^2 + \dot{z}_1 c_1 + \dot{y}_1 c_2 + k_2 z_1 + k_3\left(y_1 + x_1^2/(2r)\right) + Mg\lambda_r = -Q/r\,, \tag{9}$$

where: g – gravity constant, Q – resistance torque in the pivot S (see Fig. 1).

$$\lambda_r = \frac{\sqrt{2}l}{2r}\left(1 + \frac{y_1}{r} - \frac{y_1^2}{2r^2} - \frac{y_1^3}{6r^3}\right), \tag{10}$$

where: $l = (3\sqrt{2}/4)r$ is the distance between the centre of rotation and the centre of gravity of the rotating pendulum of mass M. The parameter z_1 and its first derivative represent internal state variables, i.e.: $z_1 = x_1 + y_1$, $\dot{z}_1 = \dot{x}_1 + \dot{y}_1$.

On that basis, the friction force in the frictional contact of the block-on-belt model follows:

$$T_S = \mu(V_r)mg\left(1 + \frac{M}{m}\lambda_r - \frac{k_3}{mg}\left(y_1 + \frac{x_1^2}{2r}\right) - \frac{c_2}{mg}\dot{y}_1\right), \qquad V_r = \dot{x}_1 - V_b\,. \tag{11}$$

In accordance to [12], the experimentally verified coefficient of kinetic friction, which is dependent on the relative velocity V_r, is proposed as follows:

$$\mu(V_r) = \frac{\mu_0}{1 + \gamma|V_r|}\left(1 + \frac{\beta}{\cosh \alpha V_r}\right)\tanh \alpha V_r\,, \tag{12}$$

where: α, β and γ are the friction law parameters controlling the shape of the curve given by Eq. (12), and $V_b = \omega_{bp} r_{bp}$ is the velocity of the conveyer belt.

3 Simulation Results

The mathematical model given in Sec. 2 by Eq. (7)-(12) has been translated on numerical procedures (virtual instruments) performed in LabVIEW. Two cases of the presented model were tested.

First case. In the simplified model, a not disturbed (constant) angular velocity of the belt pulley is assumed. Equations (1)-(4) and (7) describing the electric motor and the transmission system's dynamics are omitted.

Second case. Full model with the transmission system, of which dynamics disturbs linear velocity of the base in the block-on-belt model is assumed.

To observe differences between the simplified and full model's dynamics, their phase plane trajectories for small changes in the parameter k_3 were investigated. The control bifurcation parameter was changing from 72 to 96 [N/m] with a step of 0.001 (see diagrams in Fig. 2 and 3). During a 40 seconds simulation, the storage of time series, corresponding to each bifurcation parameter, was started after 15 seconds to omit any transitional motion.

Bifurcation diagrams shown in Fig. 2-10 exhibit relations between the coordinate x_1 of Poincaré map and the bifurcation parameter k_3. According to the assumed definition of the map, a point of phase space trajectory appears on the Poincaré map when acceleration of motion of mass *m* crosses 0 by changing its value from positive to negative.

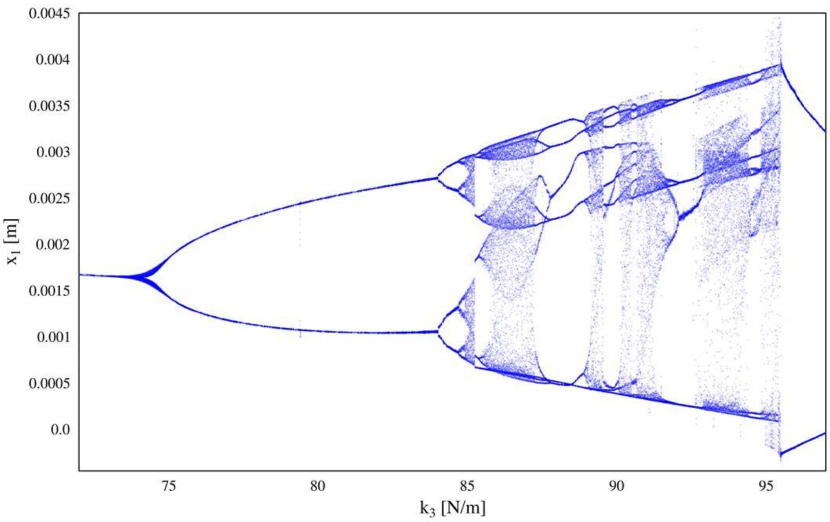


Fig. 2. Bifurcation diagram for the simplified model, where $k_3 \in [72, 97]$ (velocity of the base is not disturbed).

One observes that both models (in the first and the second case mentioned above) demonstrate significant changes in their dynamical behaviour in relation to changes in the control parameter. The main difference between the two assumed cases of complexity of the system are visible for the particular values of the bifurcation parameter k_3. In comparison to the simplified model, in the full dynamical system with the disturbed velocity of the base, the first period doublings of motion appear at lower values of control parameter. In this case, it can be observed that the regions of possible chaotic behaviour are wider.

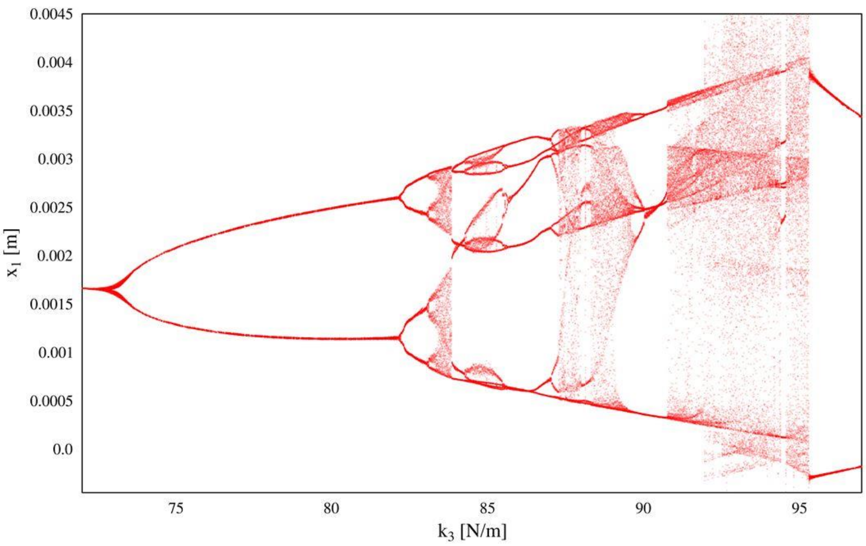


Fig. 3. Bifurcation diagram for the full model, where $k_3 \in [72, 97]$ (velocity of the base is disturbed by the driving system).

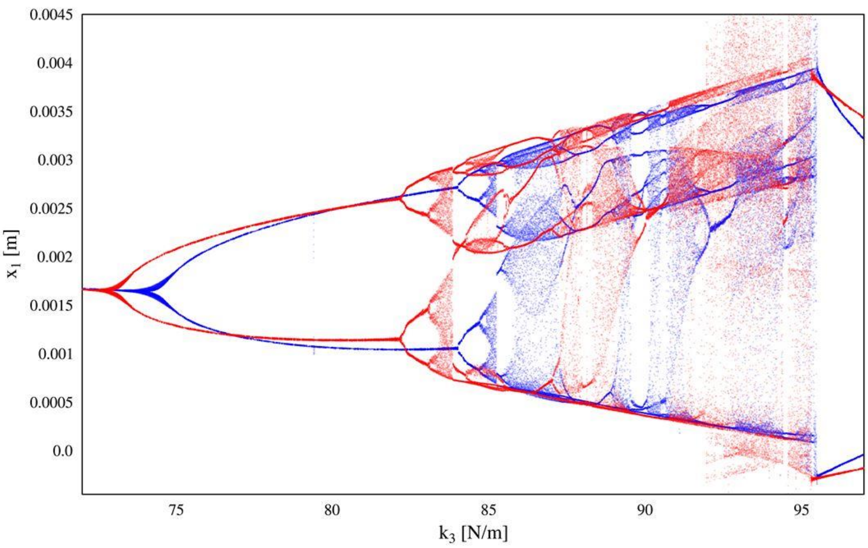


Fig. 4. Bifurcation diagrams shown in Fig. 2 (blue) and 3 (red) in the same coordinate system.

In Fig. 4, presenting two overlapped bifurcation diagrams for both analysed cases of the system's complexity, one observes, that the regions of possible chaotic behaviour end almost at the same threshold of $k_3 = 95.1$.

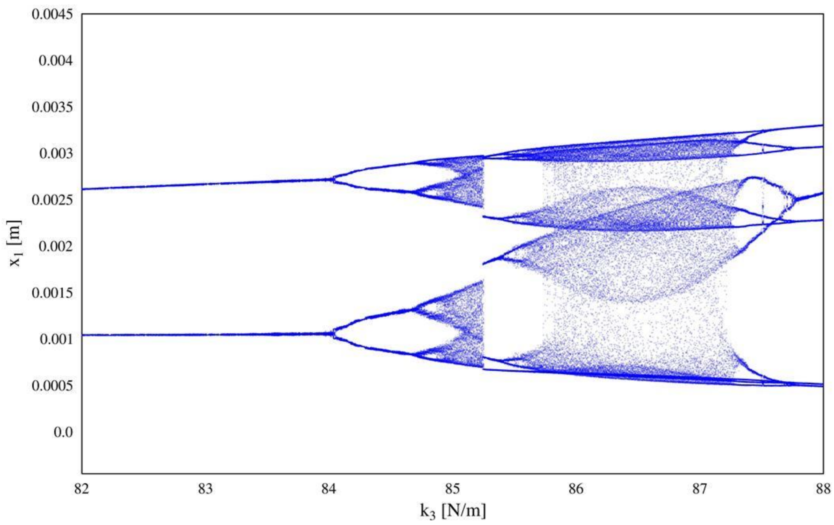


Fig. 5. Bifurcation diagram for the simplified model, where $k_3 \in [82,88]$ (velocity of the base is not disturbed by the driving system).

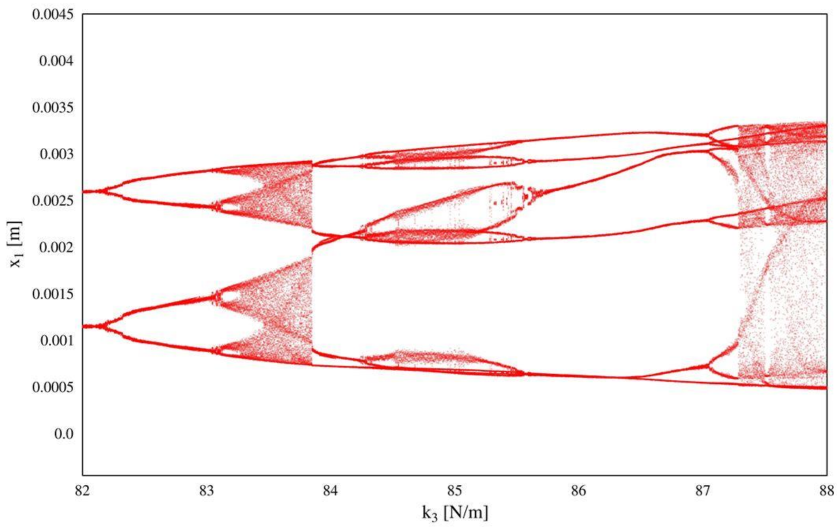


Fig. 6. Bifurcation diagram for the full model, where $k_3 \in [82,88]$ (velocity of the base is disturbed by the driving system).

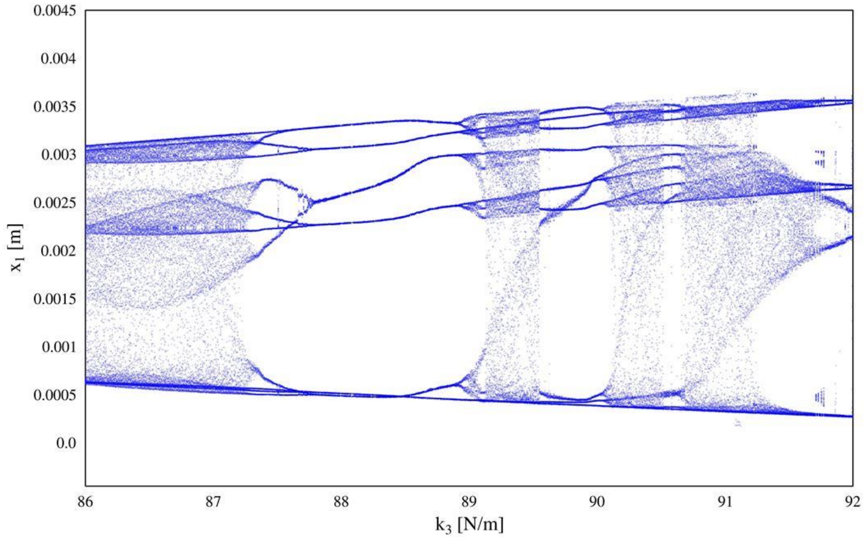


Fig. 7. Bifurcation diagram for the simplified model, where $k_3 \in [86, 92]$ (velocity of the base is not disturbed by the driving system).

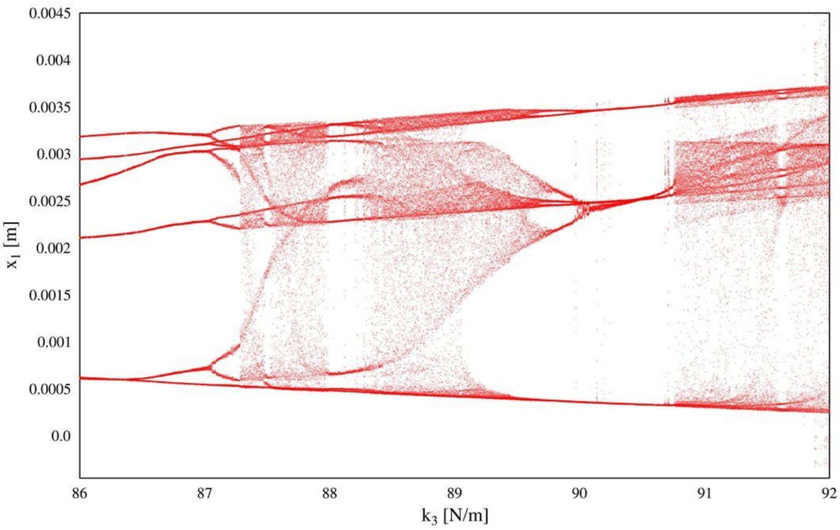


Fig. 8. Bifurcation diagram for the full model, where $k_3 \in [86, 92]$ (velocity of the base is disturbed by the driving system).

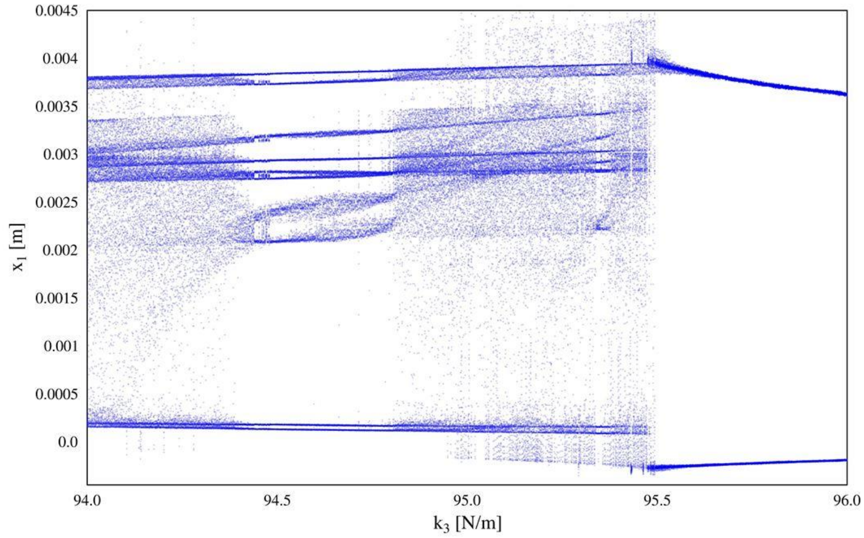


Fig. 9. Bifurcation diagram for the simplified model, where $k_3 \in [94, 96]$ (velocity of the base is not disturbed by the driving system).

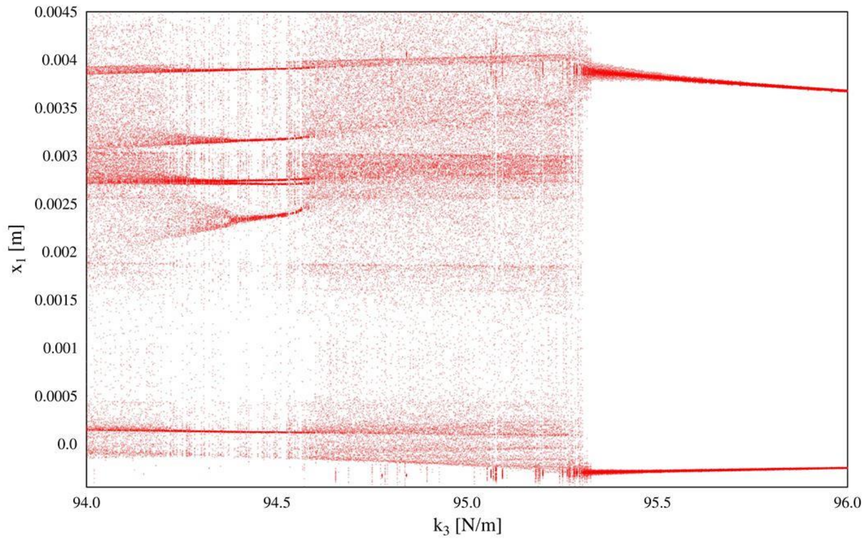
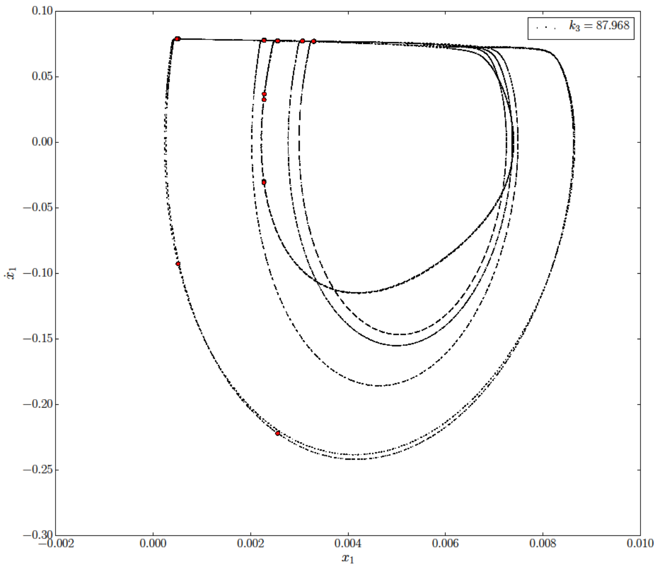
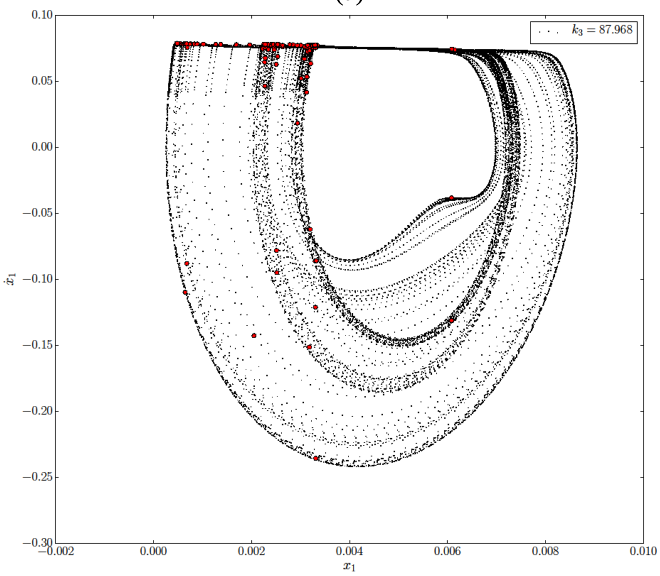


Fig. 10. Bifurcation diagram for the full model, where $k_3 \in [94, 96]$ (velocity of the base is disturbed by the driving system).



(a)



(b)

Fig. 11. Stick-slip trajectory of mass *m* and its Poincaré map on the phase plane $\dot{x}_1(x_1)$ marked by red dots of the simplified (a) and full model (b), k_3 = 87.968.

The two different solutions presented in Fig 11., i.e.: a periodic (a) and chaotic (b) obtained for the same value of parameter k_3 confirm influence of the disturbed velocity of the base on the behaviour of mass *m* on the moving base.

4 Conclusions

An influence of frequency of excitation on the dynamics of a two-degrees-of-freedom mechanical system with dry friction has been qualitatively investigated. Bifurcational behaviour of the 2-DoF system with the stick-slip effect and its response to an irregular kind of excitation caused by a disturbed velocity of motion of the base in the block-on-belt model was analysed by means of bifurcation diagrams. A mathematical model of the block-on-belt system with a normal force intensification mechanism and the DC motor based driving system with worm gear has been mathematically developed and virtualized. Although both investigated models behave similarly, exhibiting comparable changes in their dynamical behaviour, the influence of the disturbed velocity of motion of the base in the block-on-belt model is clearly visible. It has been shown that higher frequency vibrations of motors affect the driven systems dynamics.

Periodic windows on the bifurcation diagrams of the two analysed models (a simplified one and the full with the described coupling of electric drive with the driven block-on-belt transmission system) are placed in different intervals of control parameter, as well as the bifurcation branches are a bit more scattered.

As it has been observed, more scattered branches make the period-doubling bifurcations, leading to some less visible quasiperiodic or chaotic solutions, more blurred.

Basing on our results of numerical simulations presented in this work, it can be concluded, that by incorporating the dynamics of the transmission system into the block-on-belt model, even in a very basic form, a considerable impact of the source of additional high frequency oscillations on its dynamical behaviour has been proved. Therefore, if one attempts to investigate dynamics of any real system with friction, then many ideal traditionally investigated block-on-belt models with an assumption of constant velocity of the base may not properly model the real complex phenomena appearing in contact dynamics of sliding connections existing in mechatronic systems.

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