Peculiarities of Transition to Chaos in Nonideal Hydrodynamics Systems

A. Yu. Shvets[1] and V. A. Sirenko[2]

[1] National Technical University of Ukraine "KPI" , Kyiv, Ukraine
(E-mail: alex.shvets@bigmir.net)
[2] National Technical University of Ukraine "KPI" , Kyiv, Ukraine
(E-mail: sir_vasiliy@ukr.net)

Abstract. The nonideal deterministic dynamic system "tank with a fluid–electromotor" is considered. On the basis of investigation of low-dimensional mathematical model of the given system the map of dynamic regimes is constructed. The study of scenarios of transition to deterministic chaos is carried out. Atypical peculiarities of realization of such scenarios are described.
Keywords: nonideal system, regular and chaotic attractor, scenario of transition to chaos.

1 Introduction

Many of modern machines, mechanisms and engineering devices in the capacity of constructive elements contain the cylindrical tanks partially filled with a fluid. Therefore investigation of oscillations of free surface of a fluid in cylindrical tanks is one of the main problems in hydrodynamics throughout last decades [1]. Since seventieth years of past century were constructed, so-called, "low–dimensional" mathematical models describing such oscillations [2]–[5]. The "low-dimensional" models allow to obtain adequate enough describing of a problem in cases, when power of source of excitation of oscillations considerably exceeds a power consumed by an oscillating loading (a tank with a fluid). These cases are defined as ideal in sense of Sommerfeld–Kononenko [6]. However, in real practice, the power of source of excitation of oscillations more often is comparable with a power which consume the oscillating loading. These cases are called as nonideal in sense of Sommerfeld–Kononenko. In these cases it is necessary to consider interacting between a source of excitation of oscillations and oscillating loading, that leads to essential correction of mathematical models which applied in ideal cases [7]–[9].

Nonideal, in sense of Sommerfeld–Kononenko, dynamic system "tank with a fluid–electromotor" in case of horizontal excitation of a platform of tank are considered in the given article. Investigations of such systems have been begun in work [10], where the mathematical model of such systems has been

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constructed for the first time. In such model the interacting between a source of excitation of oscillations and a tank with fluid were taken into account.

The main goals of this work is detection of new peculiarities of transition to the deterministic chaos in systems "tank with a fluid–electromotor".

2 Mathematical model of hydrodynamic system "electric motor–the tank with fluid"

Let's consider rigid cylindrical tank partially filled with a fluid. We will assume that the electric motor of limited power excite horizontal oscillations of platform of tank (fig. 1). The given hydrodynamic system is typical nonideal, in sense of Sommerfeld–Kononenko [6], deterministic dynamic system. As shown in [7]–[9] mathematical model of system "tank with a fluid–electric motor" may be described by following system of differential equations:

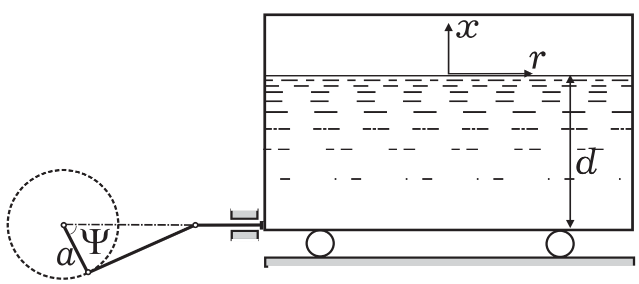

Fig. 1. The scheme of the system

$$\frac{dp_1}{d\tau} = \alpha p_1 - [\beta + \frac{A}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2)]q_1 + B(p_1 q_2 - p_2 q_1)p_2;$$

$$\frac{dq_1}{d\tau} = \alpha q_1 + [\beta + \frac{A}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2)]p_1 + B(p_1 q_2 - p_2 q_1)q_2 + 1;$$

$$\frac{d\beta}{d\tau} = N_3 + N_1\beta - \mu_1 q_1; \tag{1}$$

$$\frac{dp_2}{d\tau} = \alpha p_2 - [\beta + \frac{A}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2)]q_2 - B(p_1 q_2 - p_2 q_1)p_1;$$

$$\frac{dq_2}{d\tau} = \alpha q_2 + [\beta + \frac{A}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2)]p_2 - B(p_1 q_2 - p_2 q_1)q_1.$$

The system (1) is nonlinear system of differential equations of fifth order. Phase variables p_1, q_1 and p_2, q_2, accordingly amplitudes of dominant modes of oscillations of free surface of fluid. The phase variable β is proportional to velocity of rotation of shaft of the electric motor. There are six parametres

$A, B, \alpha, N_1, N_3, \mu_1$ of system (1), which are defined through physical and geometrical characteristics of tank with a fluid and electric motor. α – coefficient of forces of a viscous damping; N_1, N_3 – parameters of static characteristics of the electric motor; μ_1 – coefficient of proportionality of the vibrating moment; A and B–the constants which sizes depend on diameter of a tank and depth of filling with its fluid.

In works [7]–[9] existence of the deterministic chaos in system (1) has been proved, some types of chaotic attractors are classified and shown that chaotic attractors are typical attractors of the given system. We will notice that the detailed and all-round study of chaotic dynamics of system (1) is possible only by means of a series of numerical methods and algorithms. The technique of carrying out of such researches is described in works [7]–[9], [11].

3 Numerical research of dynamic regimes

Let's begin our investigations by construction the map of dynamic regimes of system. The map of dynamic regimes represents the diagram in a plane, on which coordinate axes values of two parameters of system are marked and various colors (color shades) ploted areas of existence of the various steady-states dynamic regimes. The technique of construction the map of dynamic regimes is described in [8].

In fig. 2 the map of dynamic regimes of system "tank with a fluid–electromotor" constructed in regard to parameters N_3 and α is presented at values $A = 1.12; B = -1.531; \mu_1 = 0.5; N_1 = -1$.

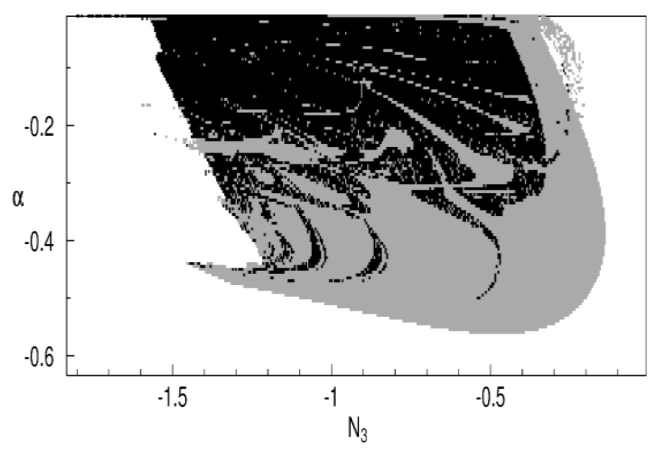

Fig. 2. The map of dynamic regimes of system.

In the received sheet of a map (fig. 2) areas of three various types of dynamic regimes are ploted. Areas of values of parameters N_3, α in which equilibrium position will be the steady-state regime of system are ploted by white color. Gray color corresponds the areas of values of parameters N_3, α at which limit cycles will be the steady-state regimes of system. At last, areas in which the steady-state regimes of system will be chaotic attractors are ploted by black color. Areas of existence of deterministic chaos (black areas) occupy the considerable space in a map of dynamic regimes. It testifies that the deterministic chaos is a typical steady-state regime of system (1).

Studying of types of the steady-state regimes of system (1) and features of realization of possible scenarios of transitions between dynamic regimes of different types we will investigate at changing of parameter N_3 along vertical section of a map (fig. 2) at $\alpha = -0.3$.

Let's consider the scenario of transition to chaos, which is realized in system at values of parameter N_3 which go out through the right boundary of a window of periodicity $-0.65269 < N_3 < -0.6296$. At each value of parameter in interval $-0.65269 < N_3 < -0.6369$ in system simultaneously exist two stable single-turn limit cycles. Their projections of phase portraits, built at $N_3 = -0.64$, are presented in fig. 3a–b. These projections are symmetrical in regard to an abscissa axis $p_2 = 0$. At parameter increasing, at value $N_3 = -0.6368$, happen a period-doubling bifurcation. In system simultaneously exist two two-turn limit cycles of the doubled period. Projections of phase portraits of cycles of doubled period at $N_3 = -0.6368$ are shown in fig. 3c–d. Projections of these cycles also are symmetrical in regard to an abscissa axis. The further increasing of value of parameter N_3 leads to arising of the symmetrical cycles of quadruple period etc. Such infinite process of periods-doubling of simultaneously existing symmetrical cycles comes to an end with arising of a chaotic attractor at $N_3 = -0.6295$ (fig. 3e–f).

The projection of the arising chaotic attractor (fig. 3e) consists of two symmetrical parts in regard to horizontal axis. Amplitudes of temporal realizations of the given chaotic attractor more than twice exceed amplitudes of temporal realizations of limit cycles of the cascade of bifurcations of period-doubling. Accordingly the chaotic attractor is localized in considerably more volume of phase space than volume of localization of any cycles of cascade of period-doubling. Moving of a typical trajectory on a chaotic attractor can be conventionally divided into two phases. In first of these phases the trajectory makes chaotic walks along coils of upper or lower parts of chaotic attractor. In an unpredictable moment of time the trajectory "jumps" from the upper or lower part of an attractor in its symmetrical part and again starts to make chaotic walks. Such process is repeated the infinite number of times. Thus transition to chaos has peculiarities which typical as for the Feigenbaum's scenario (infinite cascade of bifurcations of period-doubling of limit cycles), and as for an intermittency (an unpredictable intermittency between the upper and lower parts of arising chaotic attractor).

In fig. 4 are shown the distribution of spectrum density (Fourier–spectrums) of the constructed regular and chaotic attractors. Fourier–spectrums of single-turn limit cycles and their first bifurcation of a period-doubling (fig. 4a–b) are discrete and harmonic. It is easy to observe occurrence of a new harmonics in Fourier–spectrum in fig. 4b, that typical for the Feigenbaum's scenario. Distribution of a spectral density of a chaotic attractor at $N_3 = -0.6295$ is

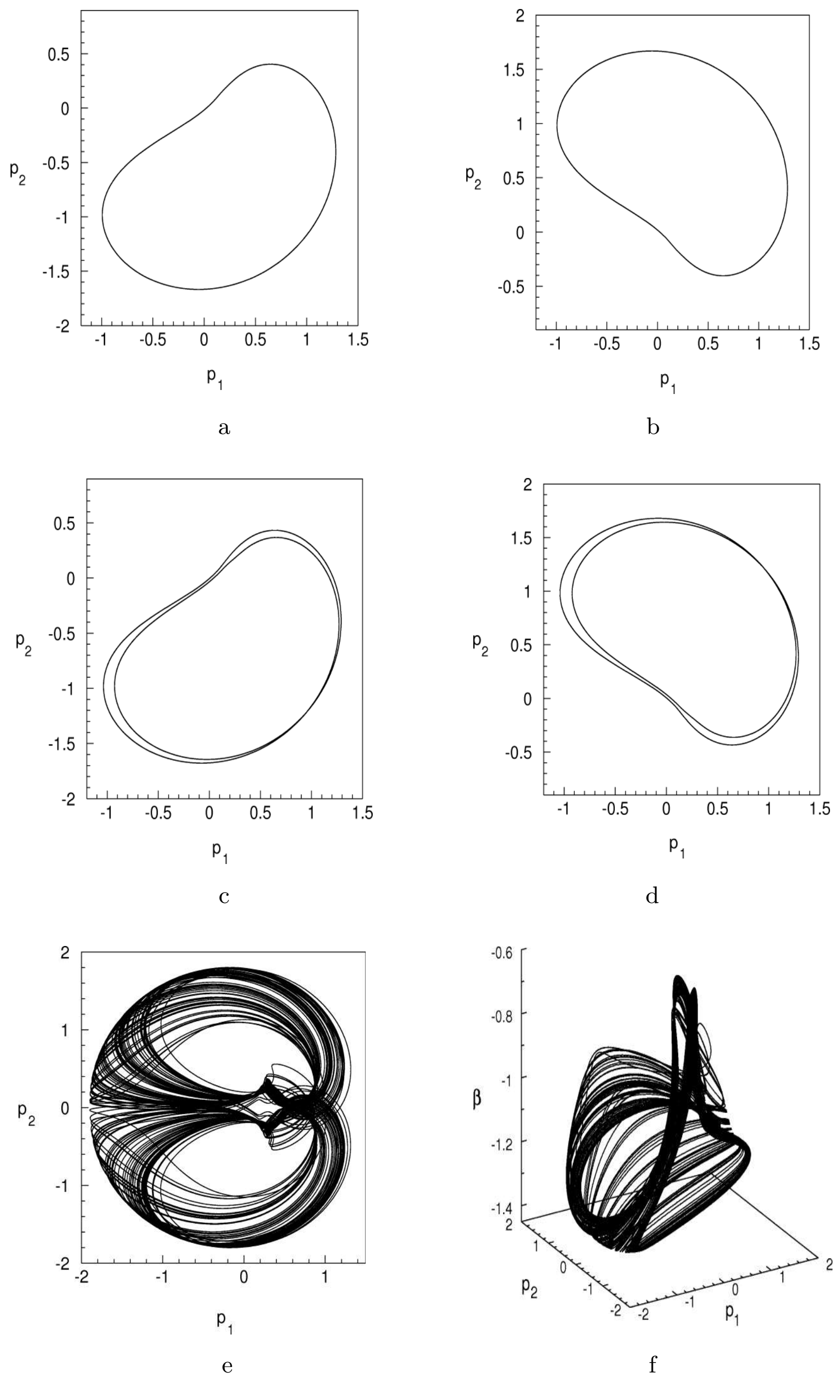

Fig. 3. Projections of phase portraits of limit cycles at $N_3 = -0.64$ (a–b), $N_3 = -0.6368$ (c–d) and chaotic attractor at $N_3 = -0.6295$ (e–f)

continuous. In its Fourier–spectrum practically completely disappear separate spectral peaks.

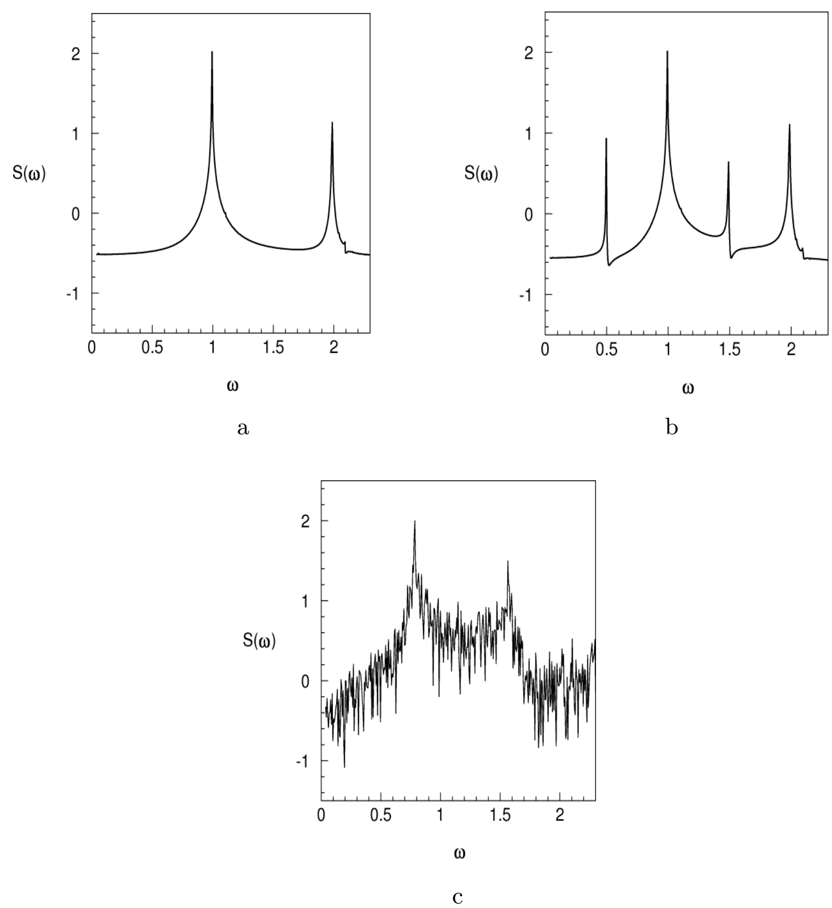

Fig. 4. Fourier–spectrum of limit cycles at $N_3 = -0.64$ (a), $N_3 = -0.6368$ (b) and chaotic attractor at $N_3 = -0.6295$ (c)

Further consider the transition to deterministic chaos through the left boundary of a window of periodicity

$$-0.65269 < N_3 < -0.6296. \tag{2}$$

As it has been told earlier, at each value of parameter in interval $-0.65269 < N_3 < -0.6369$ in system simultaneously exist two symmetrical, in regard to an abscissa axis, and stable single-turn limit cycles (fig. 3a–b). At reaching in parameter N_3 the left boundary of a window of periodicity (2), the both limit cycles are disappearing and in system arise a chaotic attractor. The projection of a phase portrait of a chaotic attractor of this kind is presented in fig. 5a. The constructed projection of this chaotic attractor is symmetrical in regard to axis $p_2 = 0$ and outwardly is similar with a projection of a chaotic attractor shown in fig. 3e.

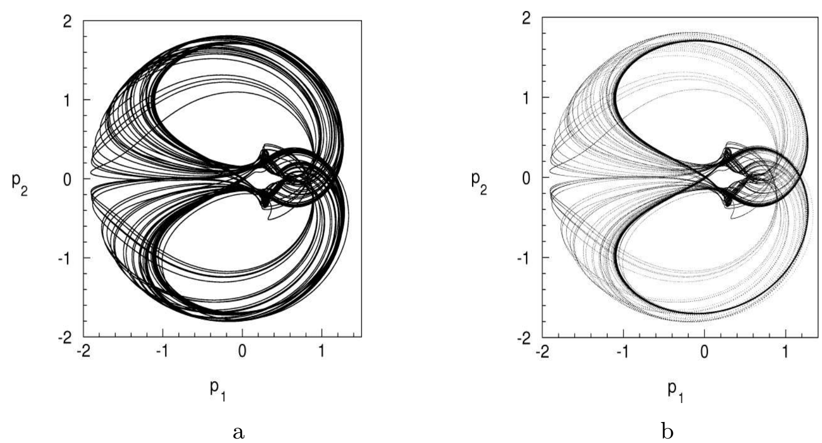

Fig. 5. Projections of phase portrait (a) and distribution of invariant measure (b) of chaotic attractor at $N_3 = -0.6527$

In fig. 5b distribution of an invariant measure in a phase portrait of a chaotic attractor is shown at $N_3 = -0.6527$. The constructed distribution makes clear the mechanism of arising of the given chaotic attractor. Contours of accurately traced area in fig. 5b under the shape represent two "pasted together" the symmetrical limit cycles presented in fig. 3a–b. Scenario of arising of chaos has many typical characteristics of an intermittency of Pomeau-Manneville. However, in this case the moving of trajectory in an attractor consists of three phases, two laminar phase and one turbulent.

In the first laminar phase the trajectory fulfils quasi-periodic motions in a small neighbourhood of one of "pasted together" disappeared cycles, either of "upper" or of "lower". In an unpredictable moment of time happens a turbulent cruption outburst and a trajectory leaves away from a neighbourhood of the disappeared cycle into distant phase space areas. To such turbulent phase of motion answer a more pale areas in distribution of an invariant measure in fig. 5b. After end of a turbulent phase, the trajectory can return into the first laminar phase of motion, or transfer in the second laminar phase, to which correspond quasi-periodic motions in a small neighbourhood of second of the disappeared limit cycles. Such process of motion of a trajectory in attractor of type "one of the laminar phases–a turbulent phase–one of the laminar phases" is iterate infinitely often. The moments of transition of trajectories into a turbulent phase, as and the moments of "switching" of trajectories between two laminar phases are unpredictable. Thus transition to chaos reminds the classical scenario of Pomeau-Manneville. However, unlike the classical scenario, we have not one, but two laminar phases.

4 Conclusions

Thus computer simulation and a numerical analysis of some aspects of the regular and chaotic dynamics of nonideal dynamic system "a tank with a fluid-electromotor" is carried out. The map of dynamic regimes of system is constructed. Atypical peculiarities of realization of scenarios of transition to deterministic chaos are revealed and described. The possibility of realization of the scenario of transition to deterministic chaos, which unites the Feigenbaum's scenario and an intermittency is detected. Also transition to chaos through an intermittency which consists not of one, but of two laminar phases is described.

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