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The phenomenon of probabilistic rotational dynamics in dynamical systems with chaotic compositional multiattractor

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Abstract: The study of motion on composite chaotic multiattractor allows to find new dynamic effects. One of them is a cyclic shifting of the region of localization of the motion in the phase space of a dynamical system multiattractor due to the asymmetry of the probability of transitions of the phase points between adjacent local attractors in closed sequences of attractors.
Keywords: dynamic chaos, generator of chaotic oscillations, composite chaotic multiattractor, replicate operator, phase cell, dynamics of active regions of attraction of phase trajectories.

A distinctive feature of the motion on the composite (composite) chaotic multiattractors [1-6] is that the chaotic motion on the local chaotic attractors, of which the composite multiattractor consists, is supplemented by random transitions of the phase point from one local attractor to another. That is, the presence of a component of motion, which could be called the dynamics of active local chaotic attractors (attractors, which at the current time is the phase point).

In the case of homogeneous multiattractors (this refers to the composite dimension of the multiattractor, equal to the number of replication variables replaced by replicating functions [7]) this part of the movement consists of transitions that can occur only in the direction of increasing or decreasing the replication variable. At the same time, if the local attractors are symmetric, not only all single transitions between neighboring local attractors are equally probable, but also all the same sequence of transitions in both directions (mirror). Therefore, in one-dimensional composite multiattractors consisting of symmetric chaotic attractors, the dynamics of the active regions of attraction is purely chaotic, and can be characterized by a number of statistical characteristics [8].

However, already in two-dimensional composite multiattractor there is another type of transition sequences – cyclic – along closed chains of local

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attractors. Therefore, in order for the dynamics of the active regions of attraction of multidimensional chaotic multiattractors to remain purely chaotic, it is necessary that the probabilities of cyclic sequences of transitions in the direction of clockwise and counterclockwise are the same.

Otherwise, there is a dynamic effect of systematic directional displacement of active local chaotic attractors along the chainThat is, the rotational self-oscillation of active local chaotic attractors.

We consider the manifestation of this effect on the example of a dynamic system, which has a composite chaotic multiattractor consisting of Lorentz attractors [9]:

$$\begin{cases} \dfrac{dx_1}{d\tau} = A\left[H_2(x_2+\mu x_1, x_1) - (1+\mu)H_1(x_1, x_2+\mu x_1)\right]; \\ \dfrac{dx_2}{d\tau} = H_1(x_1, x_2+\mu x_1)(B-x_2) - H_2(x_2+\mu x_1, x_1) + \mu H_1(x_1, x_2+\mu x_1); \\ \dfrac{dx_3}{d\tau} = H_1(x_1, x_2+\mu x_1)\left[H_2(x_2+\mu x_1, x_1) - \mu H_1(x_1, x_2+\mu x_1)\right] - Cx_3 , \end{cases} \quad (1)$$

$$H_k(x_k, x_\eta) = x_k - (d_k+1)\left\{ \frac{\left|x_k - \Delta h_k(x_\eta) - h0_k\right| - \left|x_k - \Delta h_k(x_\eta) - h0_k\left(1+\dfrac{1}{d_k}\right)\right|}{2} - \frac{h0_k}{d_k} \right\} ,$$

$$\Delta h_k(x_\eta) = \Delta_k \cdot \left[2\,sgn(x_\eta) - 1\right] ,$$

where A, B, C, D, μ – are constants determining motion on local chaotic attractors; h_k , d_k – are constants that specify the basic parameters of the replicates of the operators; H_k – the length of the phase cells by replication variables and the position of the boundary between the phase cells (h_k), as well as the width of the transition layers between the phase cells (d_k) [1]; Δ_k – is the amount of displacement of the phase boundaries of cells in *k-th* variable replication, *sgn(x)* is the Signum function [10].

Fig. 1 shows multiattractor of the system (1) $A=10$, $B=30$, $C=2.6$, $\mu=-0.2$, $h_1=17.5$, $h_2=20.8$, $d_1=d_2=30$, $\Delta h_1=\Delta h_2=0$. It is a homogeneous composite multiattractor consisting of 4 identical local attractors. When the boundaries between the phase cells are unbiased [7], the mutual transitions of motion between any pair of neighboring local attractors have an equal probability [11].

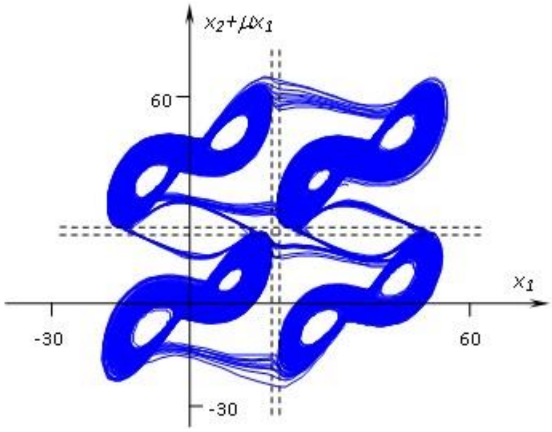

Fig.1. Multiattractor of the system (1). Dashed lines show the boundaries of the phase cells

We introduce the concept of phase index. Let us assume that its value $\Sigma\chi$ increases by one every time the phase point passes to the neighboring local chaotic attractor in the direction of rotation clockwise. And it is reduced by one if the displacement of the region of chaotic oscillations occurs in the opposite direction.

In the case where the dynamic system (1) has the parameters corresponding to Fig.1, the phase index shows systematic growth (Fig.2, case $\Delta_2=\Delta_3=0$). That is, against the background of equiprobable random transitions of motion between adjacent chaotic attractors, the position of the active local attractor constantly shifts in the direction of rotation of the clockwise.

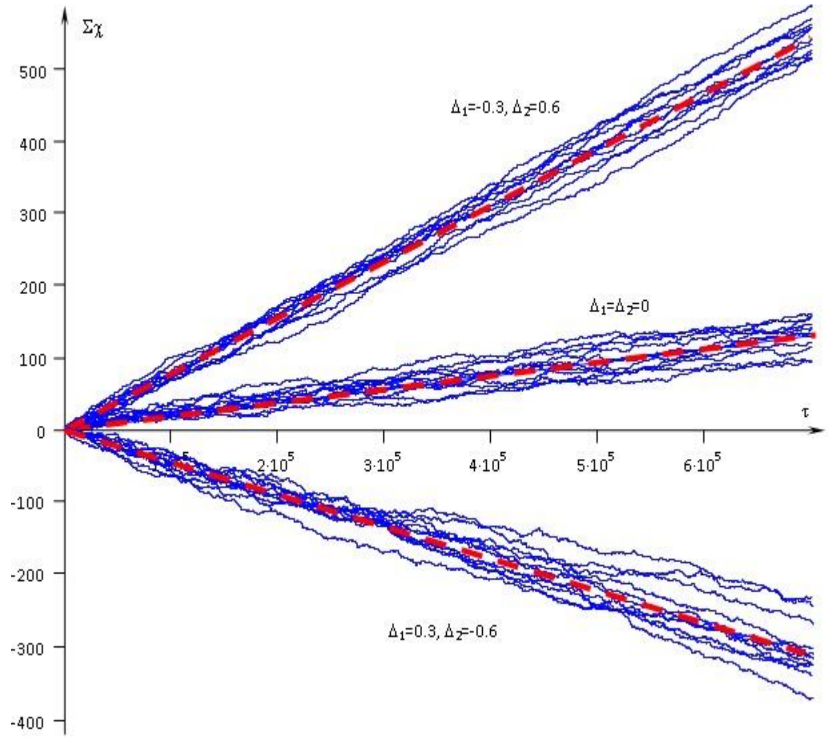

Fig.2. The dependence of the phase index on time. 11 realizations of this random process are shown for each of the 3 different positions of the boundaries between the phase cells (thin solid lines), as well as the corresponding time dependences of the mathematical expectation of this indicator (thick dotted lines).

This is because the paths that the phase point takes in successive clockwise and counterclockwise transitions are not the same (Fig.3). Taking into account the motion on attractors, this difference leads to the inequality of the probabilities of the phase point transition sequences between the local attractors in the clockwise and counterclockwise direction.

In an inhomogeneous multiattractor, the direction and speed of rotation of the active region of attraction of phase trajectories can be changed by shifting the boundaries between the local attractors relative to the equilibrium position. For example, in the system (1), choosing the values of $\Delta h_2 < 0$, $\Delta h_3 > 0$, it is possible to increase the drift velocity of the active local attractor in the direction of rotation-clockwise (Fig.2, case Δ_2=-0.3, Δ_3=0.6). By choosing $\Delta h_2 > 0$, $\Delta h_3 < 0$, the direction of its displacement can be reversed (Fig.2, case Δ_2=0.3, Δ_3=-0.6). This is achieved by creating an inequality of probabilities of mutual transitions between neighboring local attractors when the boundaries between them are shifted [11].

In particular, it is possible to find such positions of boundaries between phase cells containing local attractors of system (1), at which the systematic displacement of the active region of attraction of phase trajectories can be made arbitrarily small. Such a situation occurs, for example when Δ_2=0.1, Δ_3=-0.2.

Numerically, the rotational dynamics of active local chaotic attractors can be characterized by the average rate of increase of the phase index. For example, for those shown in Fig.2. realizations of random function $\Sigma\chi(\tau)$ it is equal to $(1.35\pm0.23)*10^{-3}$ when $\Delta_2=\Delta_3=0$, $(5.45\pm0.23)*10^{-3}$ when Δ_2=-0.3, Δ_3=0.6 (accelerated clockwise rotation) and minus $(3.08\pm0.21)*10^{-3}$ when Δ_2=0.3, Δ_3=-0.6 (counterclockwise rotation). That is, for small relative displacements of the boundaries of phase cells, the dependence of the rate of change of the phase index on the value of the displacement of the boundaries in the first approximation is linear. In this case, if we fix the ratio between Δ_2 and Δ_3 as $\Delta_3/\Delta_2=-2$, this relationship can be represented as follows: $\Omega\chi=a\Delta_3+b$, where $\Omega\chi$ is the average rate of change of the phase index, a $\approx 7*10^{-3}$, b $\approx 1.3*10^{-3}$.

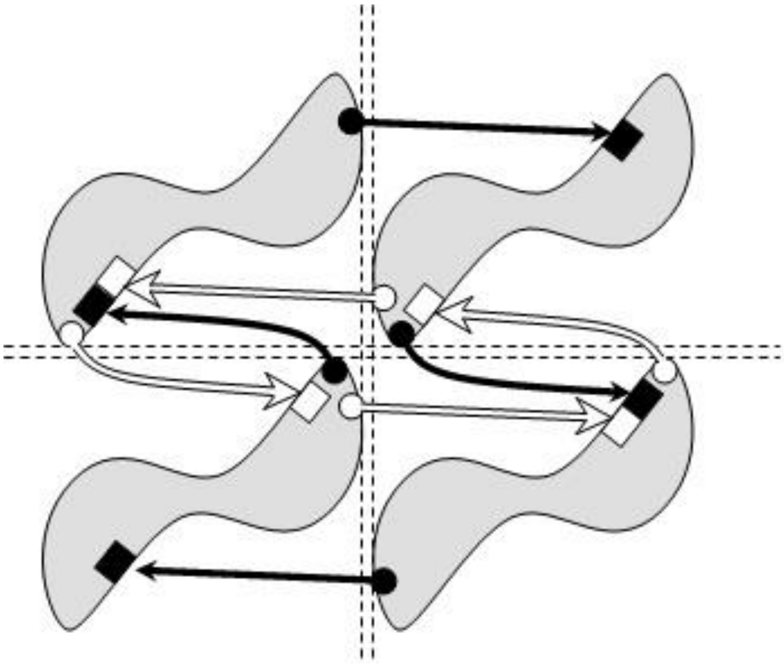

Fig.3. The scheme of phase point transitions between local attractors of the system (1) when the active local attractor is shifted clockwise (black arrows) and counterclockwise (white arrows). Circles and rectangles conditionally show the areas of attractors through which the phase point respectively leaves the attractor and falls on it during the transitions between the attractors.

Thus, in the multidimensional compositional multiattractor random transitions of the phase point between local chaotic attractors can generate a directed flow of the active domains of attraction of phase trajectories whose velocity has a pronounced deterministic component.

At the same time, in inhomogeneous QMS it is possible to change the configuration and speed of such flows by shifting the boundaries between the phase cells containing local attractors. That is, to control the effect of transformation of random motion transitions between local attractors into a deterministic displacement of the active region of attraction of phase trajectories.

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