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Multiattractor hyperchaotic system with a small perturbation of the phase trajectories

V. G. Prokopenko
Southern Federal University, Rostov-Don, Russia
E-mail: vadipro@yandex.ru

Abstract: A simple autostochastic dynamic system with a 4-dimensional hyperchaotic multiattractor is presented. The dynamics of the system differs from known analogs by small perturbation of the phase trajectories during transitions between local attractors.
Keywords: hyperchaos, compound multiattractor, hyperchaotic attractor, replication operator, dynamic system.

The construction of dynamic systems with a composite chaotic multiattractor [1,2], consisting of attractors of hyperhaotic systems [3-6], is much more difficult to build systems with a multiattractor on the basis of attractors of chaotic systems with one positive characteristic Lyapunov exponent [7-14].

This is due to the significantly more complex configuration of the regions of attraction of hyperhaotic attractors (which, consequently, are more difficult to combine in the design of a composite multiattractor). And significantly more complex nature of movements on the hyperchaotic attractors, compared with the movement on the chaotic attractors. These two difficulties create the problem of significant trajectory overrun during phase point transitions between local attractors. Which inevitably leads to considerable complication of the pattern of transitions of the phase point from one hyperchaotic attractor to another. For example, transitions occur not only between adjacent regions of attraction of phase trajectories, but also between local attractors located in phase cells that do not have a common boundary [2]. The study of such multiattractors is of particular interest [3], but no less interesting, search "classic" hyperchaotic multiattractors, in which all transitions between local attractors localized in the spaces between adjacent local attractors.

The problem of reducing the run-out of phase trajectories is, in fact, a complex one, since its solution requires taking into account several almost equivalent factors. First of all, it is an increased dimension of the phase space, since hyperchaotic oscillations can exist only in the phase space with a dimension of at least 4. Second, there is a much greater complexity of motion on hyperhaotic attractors, characterized, in general, by large deviations from the equilibrium position (hence the more significant perturbations experienced by the phase

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trajectory during transitions between local attractors). Third, a more complex configuration of hyperchaotic attractors and their areas of attraction.

The solution to this problem can be found by choosing the right replication variables [2] – to ensure optimal alignment of the regions of attraction of the phase trajectories, providing minimal perturbation of the phase trajectory during transitions between local attractors. And also to limit the energy of motion when the trajectory deviates from the shortest path between adjacent attractors.

Let us consider an example of a hyperhaotic multiattractor dynamic system, characterized by a small amount of run-off of phase trajectories during the transitions of motion from one attractor to another [4]:

$$\begin{cases} \dfrac{dx_1}{d\tau} = A\{f[H_3(x_3)] - H_4(x_4)\}; \\ \dfrac{dx_2}{d\tau} = f[H_3(x_3)]; \\ \dfrac{dx_3}{d\tau} = B[H_1(x_1) + H_2(x_2)]; \\ \dfrac{dx_4}{d\tau} = C\{H_1(x_1) + H_3(x_3) + f[H_3(x_3)] - (1+D)H_4(x_4)\}, \end{cases} \tag{1}$$

$$f(\xi) = c\xi + (a-b)\frac{|\xi+1| - |\xi-1|}{2} + (b-c)\frac{|\xi+g| - |\xi-g|}{2},$$

where A, B, C, D, a, b, c, g are constants determining motion on local chaotic attractors; h_k , d_k are constants that specify the basic parameters of the of the replication operators H_k – the length of the phase cells along replication variables and the position of the boundary between the phase cells (h_k), as well as the width of the transition layers between the phase cells (d_k) [1,2].

Figures 1-6 show the projections of the multitractor of the system (1) on the planes (x_1,x_2), (x_1,x_3), (x_1,x_4), (x_2,x_3), (x_2,x_4), (x_3,x_4), corresponding to the case when the replicating functions are defined by the equation:

$$H_k(x_k) = x_k - (d_k+1)\left\{\frac{|x_k - h_k| - \left|x_k - h_k\left(1+\dfrac{1}{d_k}\right)\right|}{2} - \frac{h_k}{d_k}\right\}, \tag{2}$$

and the constants of the system have the following values $A=2$, $B=2$, $C=2$, $D=-0.7$, $a=1$, $b=-6$, $c=0$, $g=1.9$, $h_2=2.2$, $h_3=2.4$, $d_2=d_3=10$.

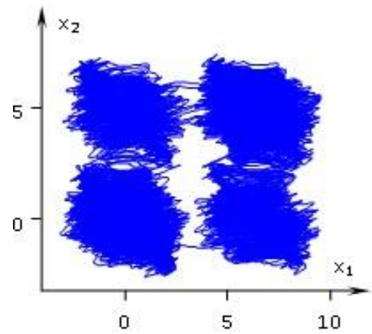

Fig.1. The projection of the multiattractor of the system (1), (2) on the plane (x1, x2).

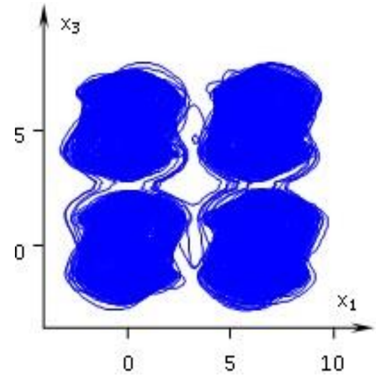

Fig.2. The projection of the multiattractor of the system (1), (2)

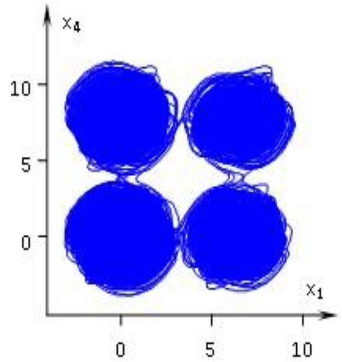

Fig.3. The projection of the multiattractor of the system (1), (2) on the plane (x1,x4).

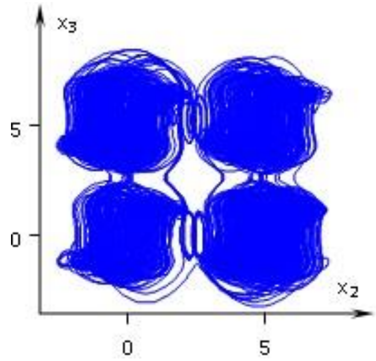

Fig.4. The projection of the multiattractor of the system (1), (2) on the plane (x2,x3).

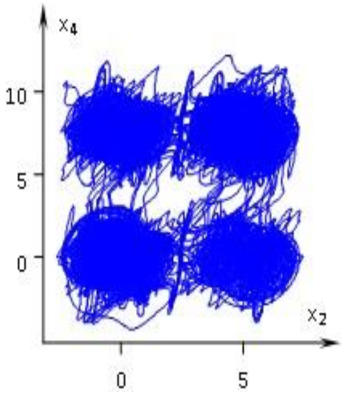

Fig.5. The projection of the multiattractor of the system (1), (2) on the plane (x2,x4).

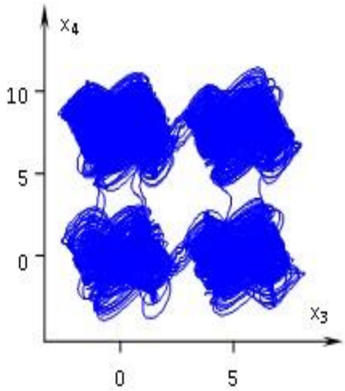

Fig.6. The projection of the multiattractor of the system (1), (2) on the plane (x3,x4).

The system (1) has a 4-dimensional hyperchaotic multiattractor in which all transitions occur exclusively between neighboring local attractors. This is the result, first of all, of the optimal mutual orientation of local attractors, as well as of the measures taken to dissipation the energy of motion when the phase point deviates from the shortest path between neighboring local attractors. In system (1) this is achieved by choosing the shape of the transfer characteristic of the nonlinear element. In addition to the sections with the opposite slope, providing self-excitation of self-oscillations and limitation their amplitude, it contains two peripheral sections with zero steepness, designed to limit the energy of motion with a significant deviation of the phase point from the nearest attractor.

In conclusion, we note that in the system (1) the phenomenon of transformation of random transitions between neighboring local attractors into different types of rotational self-oscillations of the active local attractor is observed (a local attractor on which chaotic oscillations occur at the current time). For example, in the special case of equations (1):

$$\begin{cases} \frac{dx_1}{d\tau} = A\{f[H_3(x_3)] - x_4\}; \\ \frac{dx_2}{d\tau} = f[H_3(x_3)]; \\ \frac{dx_3}{d\tau} = B[x_1 + H_2(x_2)]; \\ \frac{dx_4}{d\tau} = C\{x_1 + H_3(x_3) + f[H_3(x_3)] - (1 + D)x_4\}, \end{cases} \qquad (3)$$

there is a systematic shift of the position of the active local attractor in the counterclockwise direction. The change in the numerical characteristic of this shift (the phase index $\chi\Sigma$), which increases by one when the phase point moves from the attractor to the attractor in the direction of rotation of the clockwise and decreases by one when it moves in the opposite direction), is shown in Fig.7. Averaged over the 11 realizations of the random process $\tau(\chi\Sigma)$ given in this figure, the rate of change of the phase index is $-8\cdot10^{-4}$ with the standard deviation $2.5\cdot10^{-4}$.

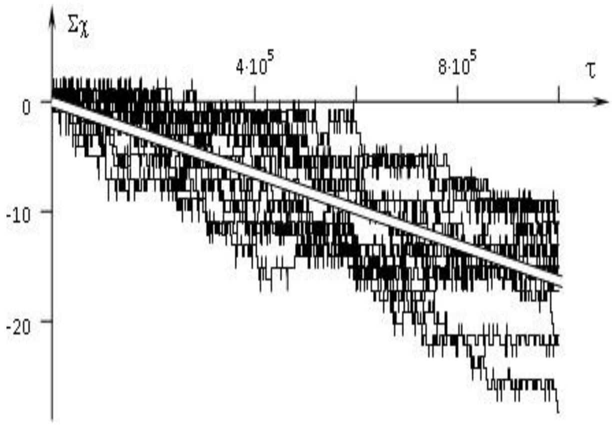

Fig.7. 11 realizations of a random process $\Sigma\chi(\tau)$ (thin broken lines) and the change of this parameter according to the average implementation rate (a wide straight line).

The expressed systematic displacement of the active region of attraction of phase trajectories is also observed in the following variants of the system (1) having a two-dimensional composite multiattractor,

$\overset{\circ}{x} = F[H_1(x_1), H_2(x_2), x_3, x_4]$, $\overset{\circ}{x} = F[H_1(x_1), x_2, H_3(x_3), x_4]$ and $\overset{\circ}{x} = F[x_1, H_2(x_2), x_3, H_4(x_4)]$.

Under the same conditions, the following values of the average rate of change of the phase index are observed in these systems: $(-4\pm4.5)\cdot10^{-4}$, $(+11.5\pm4)\cdot10^{-4}$, $(-7\pm1)\cdot10^{-4}$, accordingly.

Since the displacement of the active local attractor is nothing but the evolution of the state of the dynamic system, it can be stated that the considered system is characterized by the transformation of random transitions of the phase point between the elements of the multiattractor into a directed evolution of the state of the system as a whole.

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