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Absolute Negative Mobility in Ratchets: Symmetry, Chaos and Noise

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Abstract. Ratchet effect refers to the possibility of transporting particles in noisy systems even if the mean force is zero (zero bias). Moreover, if the transport is in the opposite direction to the bias, this transport is called Absolute Negative Mobility (ANM). In the framework of a particle in a ratchet flow, we have showed that the existence of ANM is related to a parity symmetry-breaking and to a crisis of the deterministic problem. However in the literature, ANM seems to emerge from a different scenario. It ensues the open questions of the roles of the parity-symmetry, the crisis transition and more generally the role of chaotic dynamics in the ANM phenomenon? This study provides answering elements to these questions.
Keywords: Ratchet, Absolute Negative Mobility, Chaotic dynamics, Symmetry-breaking, Noise, Particle transport, Synchronization.

1 Introduction

The control of particle transport in micro- or nano-devices has many industrial applications. Such systems are subject to non-negligible noise. These stochastic fluctuations are expected to hinder the transport. However, the noise can be a 'good friend'.
Indeed, spatial periodic systems and under a rocking force which allows a directed transport of particles is known as *ratchet* (Hänggi and Marchesoni[8]). More surprisedly, the transport may be triggered by a Gaussian noise and sometimes it is opposite to the bias (temporal mean of the forcing). In the latter case, the transport is called an *Absolute Negative Mobility* noted ANM (Machura et al.[10]).
In the literature many transport phenomena are related to the ratchet effect and ANM. Recently, Cuesta et al.[3] point out that the asymmetry plays a crucial role in the direction of the transport, in particular, in current reversal, i.e. the transport direction changes without changing the bias. They proved a very generic result for deterministic or stochastic rocking ratchets, showing that the transport velocity is a function of phase shift between the different harmonics of the rocked force. This general result may explain many current

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reversals with biharmonic forcing or asymmetric potential, e.g. Lee[9]. However, the main assumption is that the transport velocity depends only on the rocked force and not on the initial conditions. Therefore, these results cannot be applied when current multiplicity exists as in Speer *et al.*[13] or Beltrame *et al.*[2].
Recently, in the framework of particle transport in a fluid flow, called *ratchet flow*, it has be proven that the existence of ANM is generic for ratchet problems near the potential asymmetry Beltrame[1]. In particular, we show that the ANM is due to the existence of a *deterministic ANM*, i.e. a transport opposite to the bias without noise. However the scenarios described does not apply to other ratchets problems as Du and Mei[5] or Spiechowicz et al.[14]. For instance, in Machura et al.[10] there is not a deterministic ANM only a stochastic ANM is present. The goal of the paper is to clarify the existence of stochastic ANM for the 1D ratchet problems encountered in the literature. More specifically, we focus on

- The role of parity symmetry?
- Presence of chaotic dynamics in ANM existence?
- The role of the noise: does it exists a deterministic ANM?

2 Modeling

2.1 Equations

We consider the 1D deterministic transport problem:

$$\ddot{x} + \gamma \dot{x} = f(x,t) + b \tag{1}$$

where b is the bias, γ is the particle damping: large damping corresponds to small inertia. The function $f(x,t)$ is the rocking force such as:

$$f(x,t) = U(x) + R(t) \text{ or } \; U(x)R(t) \tag{2}$$

$$U(x) \text{ is 1-periodic, } R(t) \text{ is 1-periodic with zero mean value.} \tag{3}$$

This dynamics system can be expressed as

$$\dot{x} = v \tag{4a}$$

$$\dot{v} = f(x,t) - \gamma v + b \tag{4b}$$

This two-dimensional problem can be written

$$ds = (dx, dv) = F(x,v,t)\,dt = F(s,t)\,dt \tag{5}$$

The time integration of this deterministic system is performed using MATLAB. We explore the deterministic dynamics using the continuation of periodic solutions in the parameter space. This allows to track even unstable solutions and give useful informations of the dynamics. The sofware AUTO (Doedel et al.[4]) is employed.

If a perturbation due to a fluctuating force governed by a Gaussian stochastic process is added then the deterministic system Eq. 5 becomes

$$ds = F(s,t)\,dt + \epsilon\,dW, \tag{6}$$

where ϵ is an adimensional amplitude of the noise and $dW = \sqrt{dt}\,\xi(t)$ is the Gaussian distribution.

2.2 Symmetry

The deterministic ratchet problem possesses the discrete translation invariance in space and in time. Additionally, we say that f is symmetric if $f(-x, t + 1/2) = f(x,t)$. If this latter condition holds and if the bias is zero ($b = 0$) then the function F of the ODE's system Eq. (5) is equivariant by the symmetry $\mathcal{S}(s,t) = (-s, t+1/2)$:

$$F(\mathcal{S}(s,t)) = -F(s,t) \tag{7}$$

The equivariance property implies that for any trajectory $x(t)$ of Eq. (5), the dynamics $-x(t+1/2)$ is a solution too. The parity of the function is realized e.g. for $U(x)$ such as $U(-x) = U(x)$ and $R(t+1/2) = \pm R(t)$. The sign depending on the the expression of $f(x,t)$.
A solution is *symmetric* of the equivariant problem if it is invariant under the parity symmetry $\mathcal{S}$. Otherwise, it is asymmetric.

In order to better understand the role of the symmetry, we study the symmetric deterministic problem, i.e. the ODE (5) with F equivariant by $\mathcal{S}$ and then the perturbation by a bias or an asymmetry parameter. We show that the role of the symmetry depends strongly on the γ particle damping. Therefore, in the next Section we focus on large damping while in the Section 4 we consider smaller damping.

3 Large particle damping

In this part, the framework concerns the particle transport in a fluid: ratchet flow described in detail in Beltrame et al.[2]. The potential $U(x)$ corresponds to the unperturbated fluid flow without particle. Its profile is approximated by a sinusoidal variation of symmetric potential.

$$U(x) = u_m\,(1 + a\cos(2\pi x)) \tag{8}$$

A dissymmetry of this profile can be introduced using the parameter $-1/2 < d < 1/2$:

$$\begin{aligned} U(x) = u_m &+ a u_m \cos\left(\pi \frac{\bar{x}}{\frac{1}{2}+d}\right) 1_{[0;\frac{1}{2}+d]}(\bar{x}) \\ &+ a u_m \cos\left(\pi \frac{\bar{x}-1}{\frac{1}{2}-d}\right) 1_{]\frac{1}{2}+d;1]}(\bar{x}) \end{aligned} \tag{9}$$

with $\bar{x} = x \mod 1$ and 1_I is the indicator function of the interval I ($1_I(\bar{x}) = 1$ if $\bar{x} \in I$, otherwise $1_I(\bar{x}) = 0$). The parameter d shifts the relative position of the minimum (Fig. 3). Note that the potential is still unbiased when $d \neq 0$. For $d = 0$, the symmetry is recovered.

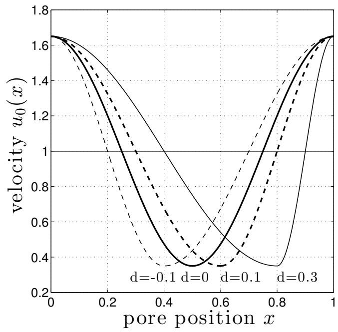

Fig. 1. Profiles of the flow $U(x)$ for $u_m = 1$, $a = 0.65$ and different values of d.

3.1 Symmetric case

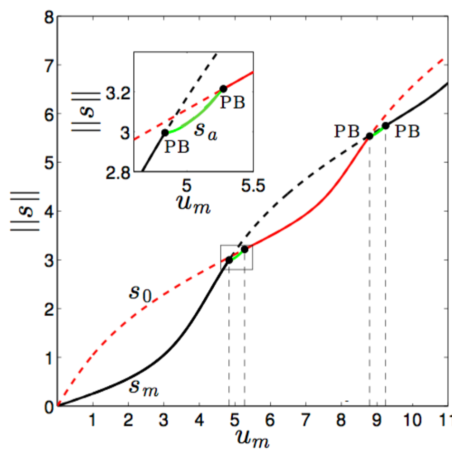

Fig. 2. Bifurcation diagram of period-one orbit branches as a function of the amplitude u_m: (black lines) symmetric branches and (green line) asymmetric branches. Bold (fine) lines indicate stable (unstable) solution branches. Dots indicate pitchfork bifurcation (PB) points. Parameters are $\gamma = 100, a = 0, 65, d = 0$ (symmetric case).

For large damping limit (γ large), the particle follows almost the fluid flow. If the amplitude of the potential is small, there is generically two solutions, s_0 and s_m with the mean position located at the extrema of the potential $x = 0$

and $x = 1/2$, respectively. Under these conditions, the solution s_m is stable while the other one is unstable [2]. By increasing the forcing amplitude, the symmetric branch s_m becomes unstable via a pitchfork bifurcation. The new branch, noted s_a, emerges supercritically and breaks the parity symmetry (Fig 2). The branch s_a connects the s_0 symmetric solution via a reverse pitchfork bifurcation and this latter gets stability. This scenario repeats when u_m further increases. The bifurcation diagrams in Figure 2 shows that there is always a stable solution s_0, s_m or s_a and the time integration shows that is the dynamics attractor. Therefore, no transport is possible.

3.2 Asymmetric case

To obtain a possible transport, we have to break the equivariance of the equation either by a bias or a dissymmetric potential, or both.
First, we consider the same symmetric potential as in the previous section and only the bias is responsible for the forced symmetry breaking. We perform the continuation of the one-periodic branch of the symmetric branches. The bifurcation diagram Fig. 3 shows that the periodic branches survive for large bias. However, at a critical value $b_c \simeq 4.54$ both branches annihilate in a saddle-node bifurcation. Note that this annihilation cannot occur between the symmetric branches s_0 and s_m for the symmetric case because the solutions have different mean values. Beyond this value an intermittent transport is observed: many particle oscillations occur in a bounded region followed by a drift to the next pore during shorter time intervals (Fig. 4). This drift is regular. The dynamics displays all characteristics of the intermittent type I bifurcation, in particular the drift velocity vanishes as $\sqrt{b - b_c}$ near the saddle-node (Manneville and Pomeau[11]). Indeed, the bias plays the same role as the asymmetry of the potential $U(x)$ in [2], and then we can show that the dynamics is similar to the phase slip in loss of synchronization for oscillator with weak forcing [12]. The time integration corroborates a quasi-periodic dynamics and that the drift velocity is independent on the initial conditions. Then, we are in the framework of the study of Cuesta et al.[3] and we expect that the direction of transport is related to the sign of the bias. The time integration confirms that for $b > 0$ the transport is always in positive direction and otherwise the transport is in negative direction. Moreover the drift velocity increases when the bias is further away from the critical bias b_c.
Analogous results occur for no-biased systems but with an asymmetric potential U: for the parameter $d > 0$, the transport is in negative direction and if $d < 0$, in the positive direction. Therefore, using asymmetry, we can counteract the effect of the bias and then construct deterministic ANM.

3.3 ANM

The loci of the saddle-nodes in the (b, d) parameters plane, displayed in Fig. 5, allows to confirm the existence of a deterministic ANM. For example, a negative transport $(c < 0)$ can be obtained with a positive bias taking a large enough

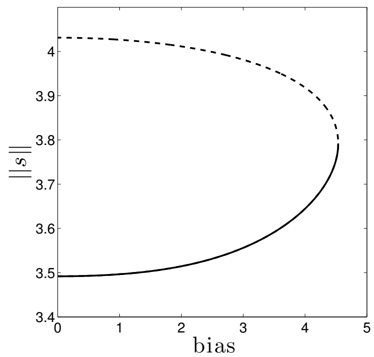

Fig. 3. Bifurcation diagram of starting from the s_0 and s_m solutions at $d = 0$. These solutions vanish via a saddle-node bifurcation. Parameters: $\gamma = 100, u_m = 6$.

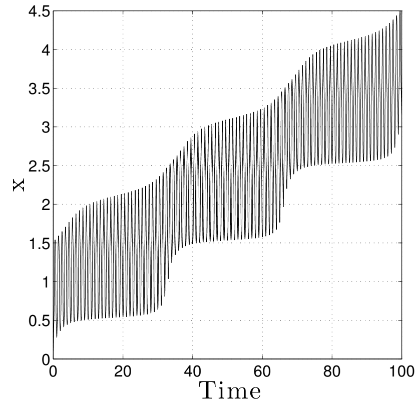

Fig. 4. Intermittent evolution of the particle for $b = 5$, where no bounded periodic solution exists.

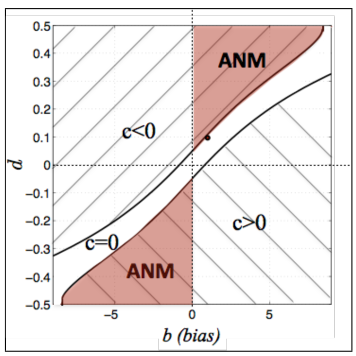

Fig. 5. Loci of saddle-nodes (black bold lines) in the (b, d) parameters plane. The saddle-nodes loci separates three regions: periodic oscillations ($c = 0$), intermittent transport either to ($c < 0$) negative and ($c > 0$) positive directions . The dot indicates the parameters for the stochastic simulations in Fig. 6. Other parameters $u_m = 9, a = 0.65$.

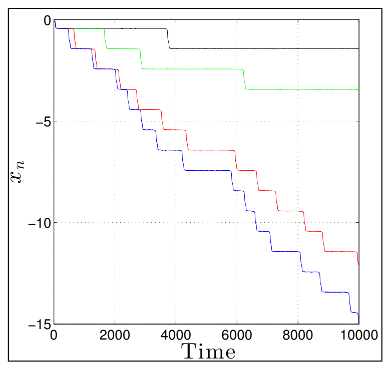

Fig. 6. Discrete stochastic dynamics $x_n = x(t_n)$ at discrete times $t_n = n$ for near the onset of the deterministic ANM (see dot in Fig 5). The different staircases dynamics correspond to different values of the noise amplitude: d=0.01 (black), d=0.02 (green), d=0.05 (red) and d=0.1 (blue). Other parameters $u_m = 9, a = 0.65$.

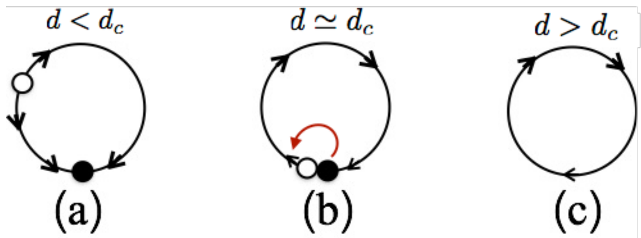

Fig. 7. Poincaré sections on the S^1 circle of the particle position $2\pi x_n$ showing the (a) subcritical, (b) critical and (c) supercritical scenarios. Full (empty) circles designate the 1-periodic stable (unstable) solution for $d < d_c$. Arrows on the circle indicate the direction of the heteroclinic connections between periodic solutions. The curved red arrow indicates a possible jump due to the noise.

dissymmetry d of the potential. Moreover, the critical dissymmetry parameter d_c is an increasing function of the bias b. For $b_\infty \simeq 8.4$ the dissymmetry reaches its upper limit $d = 0.5$ and then beyond b_∞ ANM ceases to exist. We get similar results for negative bias.

A stochastic ANM can be found by choosing the parameters near the deterministic ANM, but still in subcritical region, i.e. the deterministic dynamics is bounded. The noise allows the particle trajectory to escape from the attraction basin of the periodic orbit and to jump to the attractive manifold of the stable periodic orbit shifted by one spatial period. To understand this scenario, we have to interpret the transport transition as a loss of synchronization of a nonlinear oscillator with weak forcing [2] . In this context, the discrete dynamics x_n at entire times can be represented on a S^1 circle for which two fixed points are present in the subcritical region: the stable and unstable periodic orbits (Fig 3.3). The unstable manifolds of the unstable orbit may either connect the stable orbit in its vicinity or the stable orbit shifted of one spatial period. At the saddle-node bifurcation, both periodic orbits collapse and a heteroclinic orbit connects the shifted collapsed periodic orbits. In the supercritical region the quasi-periodic dynamics occurs via a rotation. If we are close to the saddle-node bifurcation, a small noise allows the trajectory close to the stable periodic orbit to jump close to the unstable manifold which leads to the shifted stable orbit (see red arrow in Fig. 3.3). Then the dynamics will probably drifts to the next stable periodic orbit. Note that the probability to drift in other direction is small because of the asymmetry of the deterministic dynamics.

The stochastic simulation corroborates this scenario. We depict single realizations of the stochastic discrete dynamics at entire periods (Fig. 6) for different amplitudes of the noise. The discrete trajectories display long plateaux which correspond to dynamics near the periodic orbit interspersed by short drift to the shifted periodic orbit. This latter transition corresponds to a trajectory connecting the unstable periodic orbit to the next shifted stable orbit. The mean drift velocity increases with the amplitude of the noise which is consistent with the fact that larger amplitude of the noise increases the probability to escape from the stable periodic orbit (see red arrow in Fig. 3.3). Note that

the dynamics is similar to the deterministic transport in the supercritical region, but in the deterministic case, the dynamics is quasi-periodic and then the plateau lengths are regular contrary to the stochastic case.
This ANM scenario allows to explain simulations done in literature. For instance, Lee[9] presents stochastic drifts of overdamped particles. The simulation displays similar behaviors as in our current study, however the responsible transitions and mechanisms are not explained. In another context, Guo and Diamond[7] shows the triggering of the phase-slip of H-mode in tokamaks via the noise. We believe that the mechanism is similar to the present paper since our dynamics can be interpreted as a loss of synchronization.
In conclusion, we found out a mechanism of ANM for large damped particles. This mechanism requires an asymmetry to counteract the bias and to obtain a deterministic ANM. The noise allows to trigger the deterministic ANM.

4 Moderate particle damping

The dynamics for small or moderate damping displays more complex bifurcation scenarios especially route to chaos. The description of the scenario was done in Beltrame et al.[2] in the framework of ratchet flow. We deduced a scenario of ANM in this framework (Beltrame[1]). However, some ANMs reported in literature does not seem to fit to this scenario. We choose the model of Machura et al.[10] to much deeper understand this new scenario. In this case, the rocking force $f(x,t)$ is given by

$$f(x,t) = \cos(2\pi x) + a\cos(2\pi t) \qquad (10)$$

then without bias, the problem (4) is equivariant under the $\mathcal{S}$ parity symmetry. The damping is small $\gamma = 0.9$ and the control parameter is a.

4.1 Deterministic dynamics

Symmetric case We retrace the scenarios from the 1-periodic solutions synchronized with the rocking force. As in the bifurcation diagram Fig. 2, the symmetric solutions lose or gain stability via pitchfork bifurcations. The s_a asymmetric branch connects the two pitchfork bifurcations (Fig. 8). However, this time the s_a branch becomes unstable via period-doubling bifurcations near the pitchfork bifurcations. The period-doubled branch connects the other period-doubling bifurcation on the s_a and it is unstable in a large range because of period-doubling bifurcations (Fig. 8). Indeed, a cascade of period-doubling occurs in a finite range according to the Feigenbaum law (Tresser and Coullet[16] and Feigenbaum[6]). Then a period-doubling route to chaos follows the same kind of scenarios as described in Seer et al.[13] and deeper analysed in Beltrame et al.[2]. We recall briefly the scenarios of the paper [2] in the framework of the ratchet flow with $\gamma = 10$. From the period-doubling cascade emerges a chaotic dynamics but the strange attractor remains bounded. Due to the periodicity of the system there is a series of shifted strange attractors. Increasing further the control parameter, a merging crisis occurs between the

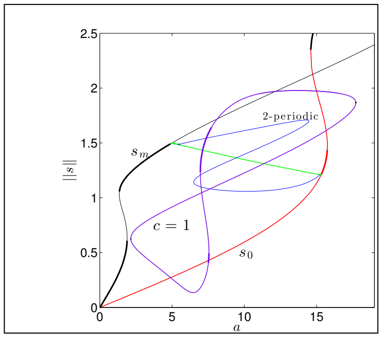

Fig. 8. Bifurcation diagrams of periodic orbits for the problem given by the rocking force (10) without bias (symmetric system): symmetric solutions s_0 (red), s_m (black), asymmetric solution s_a and 2-periodic solution (line in blue). Synchronized transport with $c = 1$ corresponds to the magenta line. Bold line indicates stable solutions while fine line indicate unstable.

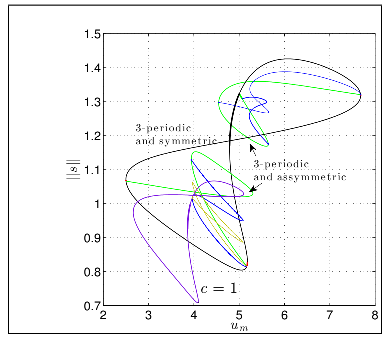

Fig. 9. Bifurcation diagrams of 3-periodic orbits and doubled periods: (black) 3-periodic and symmetric branch, (green) 3-periodic asymmetric branch, (blue) 6-periodic branch, (brown) 12-periodic branch. The magenta branch is the $c = 1/2$ synchronized transport. It occurs in the chaotic dynamics regions resulting from period-doublings.

contiguous attractors. The resulting attractor is no longer bounded since all strange attractors merge. It results a chaotic and intermittent dynamics which is not bounded. Because of the parity symmetry, there is no net transport, the dynamics resembles anomalous diffusion. Increasing further, a synchronization transition occurs: a 1-periodic transport occurs with the velocity $c = 1$. This solution emerges and ends at saddle-nodes. In its existence domain, the velocity is *locked* as in synchronization of oscillators with large driving (Pikovsky et al.[12]). Near the onset of synchronization, the dynamics is intermittent and chaotic too.

In the framework of Machura et al.[10], the scenario is similar. However the emergence of the synchronized transport with $c = 1$ is beyond the chaotic attractor (Fig. 8). Moreover, the stable 1-periodic solution hides the possible chaotic dynamics near the emergence of transport and then we are not able to observe the chaotic transition. We believe that this emergence occurs subcritically. Indeed, in the framework of [2], we remark that the transition becomes subcritical when γ decreases.

In this range $a \in [2; 8]$, there is another scenario which leads to synchronized transport with $c = 1/2$, i.e. during two temporal periods the particle advances from one spatial period. This scenario involves a symmetric 3-periodic solution which emerges and ends at saddle-nodes (Fig. 9). The existence of this solution may come from chaotic sets which confirms the existence of chaotic dynamics in this range. This symmetric solution becomes unstable via pitchfork bifurcation leading to asymmetric 3-periodic solutions. Period-doubling bifurcations arise on these latter solution branches. Note that the period-doubling bifurcation cannot occur on the 3-periodic symmetric branch according to Swift and Wisenfeld[15]. This route to chaos via period-doubling leads to the same scenario as previously: a merging crisis generates an unbounded dynamics following by a synchronization leading to $c = 1/2$ synchronized transport. We shown already in Beltrame et al.[2] that this specific velocity value is due to the strange attractors born by the period-doubling cascade of the 3-periodic solutions. The synchronized transport is almost unstable expect in a small range near $a = 4$. In this range, an symmetric 1-periodic stable solution coexists.

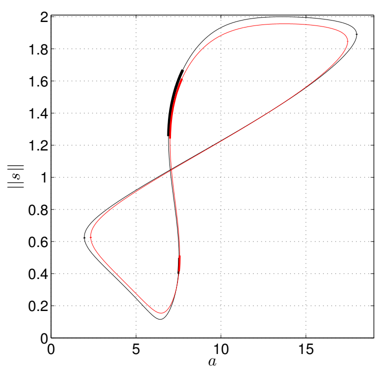

Fig. 10. Synchronized transport solutions for a bias $b = 0.1$: (black) $c = 1$ and (red) $c = -1$. Bold lines indicate stability.

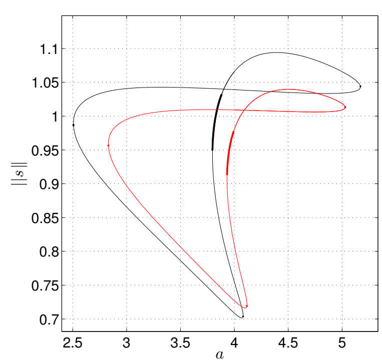

Fig. 11. Synchronized transport solutions for a bias $b = 0.1$: (black) $c = 1/2$ and (red) $c = -1/2$. Bold lines indicate stability.

Asymmetry perturbation The synchronized transport $c = 1$ or $c = 1/2$ in the symmetric case implies the existence of their symmetric transports, i.e. $c = -1$ and $c = -1/2$. Generically a small perturbation of the parity symmetry, i.e. $b \neq 0$ or $d \neq 0$ will not destroy the solutions. However their existence domain and its stability domain differ between opposite transport solutions (Figs. 10-11). Note that the existence domain of the positive transport (in the same direction of the bias) overlays the existence domain of the negative transport. In this sense, the bias 'promotes' the transport to its direction. However, when both transport solutions are unstable, the resulting dynamics is complex and the action of the bias can lead to deterministic ANM. We study it in the next subsection.
The forced symmetry breaking allows a net drift near the emergence of the synchronized transport, too. Indeed, as for the symmetric case, there is an unbounded chaotic dynamics. However, contrary to the symmetric case, the drift velocity is generically non-zero because of the lack of parity symmetry, e.g. see Beltrame[1].

4.2 Scenarios of ANM

According to the previous subsection, the add of a small bias implies that transport solutions in opposite directions exist for different ranges and have different stability domains. This fact shows that generically we may obtain a transport direction opposed to the bias, in other words, a deterministic ANM. We describe two scenarios based on this idea.

ANM via chaotic dynamics This scenario is presented in Beltrame[1] in the ratchet flow framework. When the bias is non-zero a chaotic dynamics exist near the emergence of the synchronized transport. However the transport opposite to the bias is not necessary an attractor. The trick is to tune the dissymmetry parameter d so that this transport becomes an attractor. Then, applying a bias, small enough, this attractor remains and we obtain a deterministic ANM. The stochastic ANM is obtained for parameters near the emergence of this chaotic transport. In this subcritical region, the strange attractor is bounded. Because of the periodicity, a series of shifted attractors are present. Small fluctuations allows to jump from an attractor to a consecutive one and a non-nounded dynamics occurs. Finally, because of the asymmetry of the system, a net drift appears (see Fig. 8 in [1]).
In this scenario, this ANM requires the existence of a near symmetric case and occurs in a chaotic dynamics.

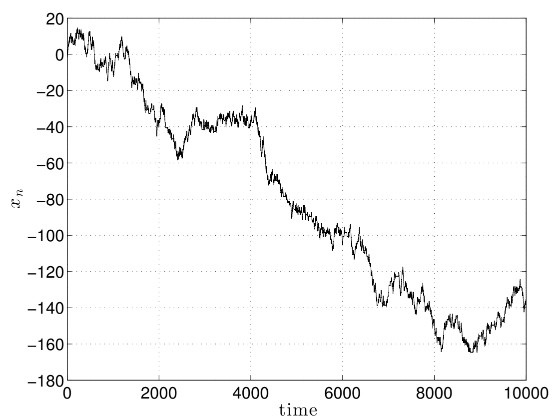

Fig. 12. Discrete stochastic time evolution $x_n = x(n)$ in the framework of [10] with $a = 4.2$ and $\epsilon = 0.05$.

ANM without chaotic dynamics The ANM studied in Machura et al.[10] occurs for $a = 4.2$ and the bias $b = 0.1$. According to the bifurcation diagram Fig. 8, the 1-periodic solution is the unique attractor. By applying, a small noise the dynamics seems chaotic with displacements in both directions. However it displays a small negative drift: see a realization of this stochastic dynamics in Fig. 12. The authors point out the presence of trajectory near the synchronized transport $c = -1/2$. However, according to the bifurcation diagrams of Figs. 8 to 11, there is many unstable dynamics and in particular the transport $c = +1/2$. Indeed, we can also observed events of transport near

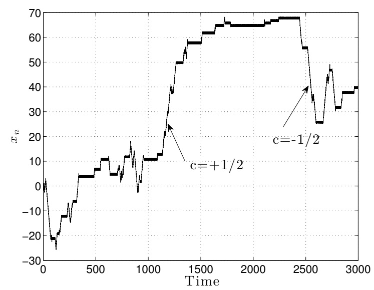

Fig. 13. Discrete stochastic time evolution for $a = 3.96$, $b = 0.1$ and $\epsilon = 0.05$. The noise destroys the deterministic ANM.

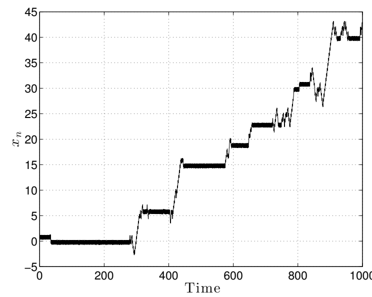

Fig. 14. Discrete stochastic time evolution for $a = 3.80$, $b = 0.1$ and $\epsilon = 0.05$. The noise induces a positive mobility.

the synchronized transport $c = 1/2$. Note that by slightly increasing the parameter a, the ANM disappears and a transport to positive direction appears. Therefore, for this scenario the role of the noise is difficult to predict because opposite transport solutions are in competition.
Moreover, we can find an example where the noise destroys the deterministic ANM. For example, at $a = 3.96$ the negative transport $c = -1/2$ is stable. Then the deterministic dynamics is either attracted by the 1-periodic bounded dynamics or by this transport. We obtain for some initial conditions a deterministic ANM. If we apply a small noise, the dynamics can escape from the ANM dynamics and we observe events of $c = +1/2$ (Fig. 13). The resulting dynamics does not display a net drift. Thus, the noise 'kills' the non-linear ANM effect.
Still in the vicinity of the latter parameters, the noise may have a contrary effect that shown in Machura et al.[10]: trigger a **positive** mobility. For $a = 3.8$, the 1-periodic bounded orbit is the unique attractor again. By adding a noise, we observe a net positive drift (Fig. 14). The dynamics is intermittent with long stays near the 1-periodic orbit and then events of positive transport essentially near $c = 1/2$. The value $a = 3.8$ is close to the saddle-node of the solution $c = 1/2$ (see Fig. 11) where a stable solution emerges. We believe the existence of a chaotic set corresponding to a positive transport near the saddle-node. But, this chaotic set appears to be a repellent and it is not observable by time integration simulation. However, the noise allows to escape the 1-periodic attractor and to visit this chaotic dynamics intermittently.

In a general way, to better understand the rich variety of noise effects, we have to assume the existence of non-attractive chaotic dynamics in this parameter region. Indeed, when no stable periodic orbit hides the dynamics in range of unstable synchronised transport direction, a chaotic dynamics leading to a net drift is observed (see Seer et al.[13] and Beltrame et al.[2,1]). Moreover, the drift velocity changes by slightly tunes the parameters. In particular the drift direction. We believe that such chaotic drift is present but it is non-attractive or its attraction basin is very narrow. The direction of this unstable

chaotic deterministic drift will determine the stochastic drift direction and then the possible ANM.
In this framework, the prediction of the ANM requires the knowledge of chaotic sets and even chaotic repellent.

5 Conclusion

In this study, we clarify the conditions of the emergence of ANM in 1D ratchet problems. Only in narrow parameter ranges, the noise may induce ANM. The determination of these ranges requires the knowledge of the dynamics in the phase space, not only attractors but also unstable sets as unstable periodic orbits or chaotic repellents.
The ANM mechanisms depend strongly on the order of γ damping magnitude. For large damping, the asymmetry of the potential is crucial and allows to counteract the natural effect of the bias. Then, if the asymmetry is large enough we may get a deterministic ANM. The stochastic ANM is obtained for parameters close to the transport onset and the resulting ANM is intermittent and slow.
If the damping is no more large, additionally non-linear effects appear and, in particular, chaotic dynamics. The consequence is the existence of stable transport solutions even in the symmetric case. Noteworthy, the bias (if small enough) triggers the existence of deterministic ANM in a parameter region. Indeed, the bias acts essentially as a forced symmetry-breaking. Its natural effect, i.e. transport in its direction, occurs in linear systems but it can be destroyed by the non-linear dynamics. Then, the dissymmetry is a necessary ingredient of the ANM but contrary to the previous scenario it has to remain small. We present two cases. The first one involves a merging crisis. The noise allows to trigger the crisis as for the scenario involving large damping. Therefore, the direction of the transport is induced by the deterministic dynamics in the supercritical region. The ANM proposed by Machura et al.[10] does not involve such deterministic ANM: without noise the dynamics converges to a 1-periodic solution. However, we believe that an unstable deterministic ANM is present due to a chaotic repellent. Without the knowledge of the chaotic dynamics, the effect of a small noise is difficult to be predicted: it can even 'kill' a deterministic ANM.
Therefore, the role of chaos in ANM depends on the value of the damping γ. For large γ there is no chaotic dynamics involved in ANM. In contrast, for γ about 10 or smaller the chaotic dynamics is present and its knowledge is required to predict and control ANM induced by noise. The role of symmetry depends on these both classes: for large damping, the asymmetry of the potential is required and has to be large enough, in contrast for for moderate and small damping, only the bias is required and the ANM can be improved by a small dissymmetry of the potential.

Even if the present study focussed only on the simple 1D ratchet problem governed by a system of ODEs, we believe that such a generic scenarios may apply for 2D or 3D ratchet problems governed by a system of PDEs. The advantage of our approach is to give a systematically way to find out the possible ranges of ANM only by using continuation methods of periodic orbits of the deterministic ANM. That has considerable gain of CPU time comparing to stochastic realisations for different parameters in order to capture a possible ANM effect.

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