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One of the Simplest Chaotic Generator: Modeling, Research and Control

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Abstract. In this paper one of the simplest chaotic circuits is presented. The circuit was modeling by using MultiSim software environment. System's behavior is investigated through numerical simulations, by using well-known tools of nonlinear theory, such as phase portrait, bifurcation diagram, Lyapunov exponents and Kaplan–Yorke dimension. Submitted by chaotic attractor and time distributions of two chaotic coordinates. Submitted values of capacitor C2 in which generated controlled chaotic attractors. Control 2-period and 3-period attractors and time distributions of two chaotic controlled coordinates are also submitted. For the first time was conducted control CCC circuit. For demonstration was applied modern software environment MultiSim. The values of capacitor C2 can be used as keys for masking and decryption of information carrier in modern systems transmitting and receiving information.
Keywords: simplest chaotic generator; control; MultiSim.

1 Introduction

The design of chaotic oscillator circuits has been a subject of increasing interest during the past few years because of the possible applications of chaos in several areas and particularly in communication. As an active topic of research, it has advanced significantly due to the pioneering contributions made by many authors. The main thrust of this research is to discover new chaotic oscillator circuits and to further study the dynamics responsible for generation of chaos in these circuits. In the past three decades a variety of nonlinear electronic oscillator circuits consisting of either real nonlinear physical devices such as nonlinear diodes, capacitors, inductors and resistors or devices constructed with ingenious piecewise linear circuit elements have been utilized as veritable black boxes to explore different properties of chaotic dynamics.
The generation, control and application of chaotic attractors have been studied with increasing interest and have become a central topic in research due to its great potential in chaos communication technology [1-4].

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Chaos theory has been established since the 1970's due to its applications in many different research areas, such as electronic circuits [5, 6], secure communication systems [7-10], magnetism [11], economy [12] etc.
A great interest is the simulation that using different software environments allows to demonstration different information properties of chaotic oscillations [13].
In recent years, there has been intense research activity devoted to the design of effective control techniques. A large body of work derives from the Ott, Grebogi, and Yorke (OGY) idea [1], which seeks to use small perturbations to place chaotic orbits onto unstable periodic orbits.

2 CCC generator

Many circuits realize chaotic generators. In many cases is a basic element of the functional blocks of chaotic secure communication systems.
One of these circuits is capacitor chaotic circuit (CCC) that described of equation [14]:

$$\dddot{x} + A\ddot{x} + B\dot{x} = C\left(\operatorname{sgn}(x) - x\right) \tag{1}$$

where the signum function $\operatorname{sgn}(x)$, which can be conveniently approximated using an op-amp without negative feedback, is defined as

$$\operatorname{sgn}(x) = \begin{cases} -1, & for\ x < 0, \\ 0, & for\ x = 0, \\ 1, & for\ x > 0. \end{cases} \tag{2}$$

Using the phase variable $x = [x_1, x_2, x_3]^T$, we can rewrite the system equation (1) as

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -C & -B & -A \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ C\operatorname{sgn}(x_1) \end{bmatrix}. \tag{3}$$

For the purpose of analysis, (3) can be divided into three linear regions, which are piecewise continuous. Specifically, we consider an inner region arbitrarily close to the origin (where $|x_1| < \varepsilon$ for some arbitrarily small positive constant ε) and two outer regions where x_1 is well away from the origin. In

the outer regions, the term $C\,\mathrm{sgn}(x_1)$ is simply a constant. Thus, the outer regions are defined by two affine equations with the same Jacobian as (3), and they have the same eigenvalues.
In the inner region, we connect the two outer regions with a straight line, so that

$\mathrm{sgn}(x_1) - x_1 \approx x_1/\varepsilon$ for $|x_1| < \varepsilon$.

Therefore, in this inner region, the constant term disappears, and the system equation is

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ C/\varepsilon & -B & -A \end{bmatrix} x . \tag{4}$$

In the circuit implementations, A, B, and C are functions of the passive components, which all have positive real values.
Figure 1 shows simulated scheme of one of the simplest chaotic generator by using MultiSim. This scheme consists one active integrator (R_C-R-C1-U1A) and a passive second-order integrator (R1-C2-R2-C3).

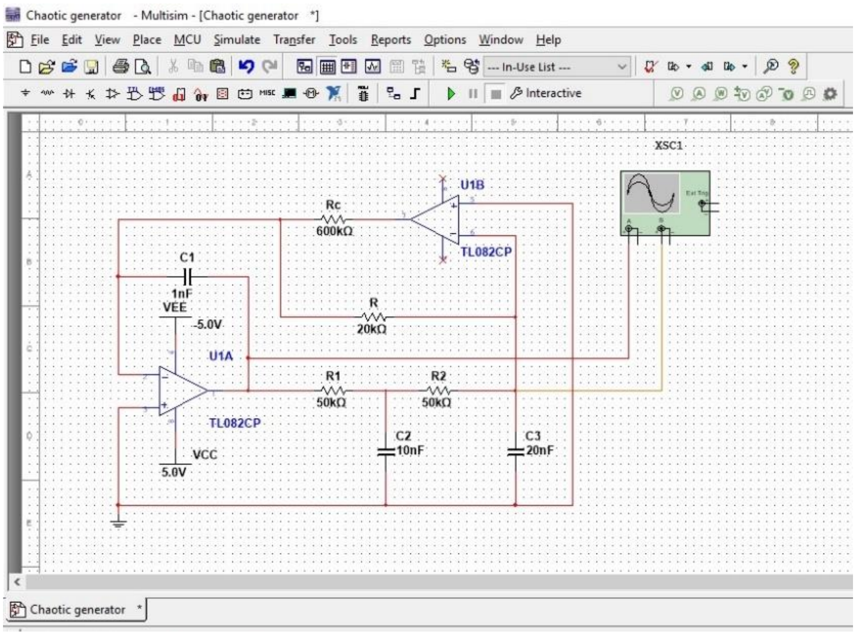

Figure 1. Simulated circuit of CCC generator

Circuit solves (1) with the parameters given by

$$A = \frac{R + R2}{RR2C3} + \frac{R1 + R2}{R1R2C2}$$
$$B = \frac{R + R1 + R2}{RR1R2C2C3} \tag{5}$$
$$C = \frac{1}{RR1R2C1C2C3}$$

Circuit was realized on the one operational amplifier TL082, powered by a 5V, one diode 1N4148, resistors R_C = 600 kΩ, R = 20 kΩ, R1 = R2 = 50 kΩ, capacitors C1 = 1 nF, C2 = 10 nF, C3 = 20 nF.

Figure 2 shows the result of circuit simulation. Generated chaotic signal in the plane XY presented on the virtual oscilloscope. Coordinate X in the circuit correspond output of the operational amplifier U1A, coordinate Y – the output of operational amplifier U1B.

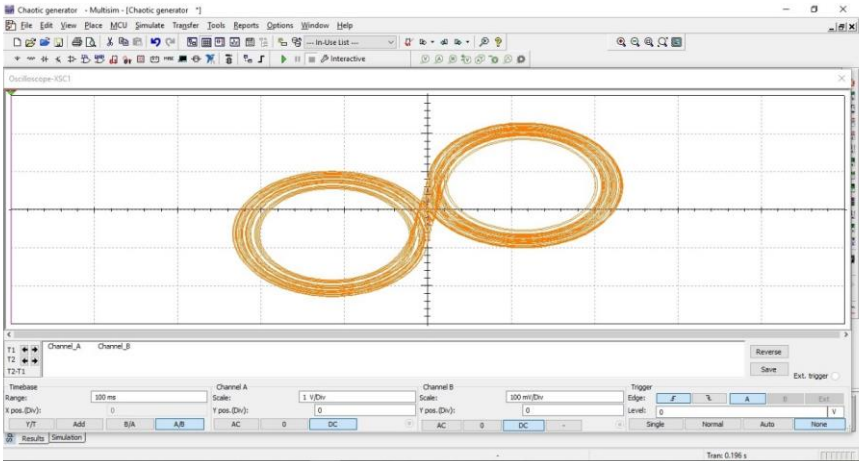

Figure 2. Chaotic attractor that generated CCC generator

Figure 3 and Figure 4 show time dependences of the coordinates X and Y respectively.

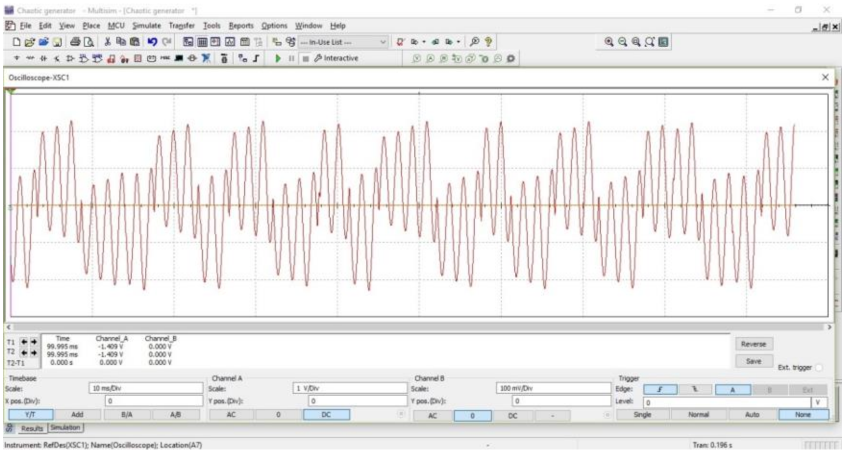

Figure 3. Time dependence of the coordinate X

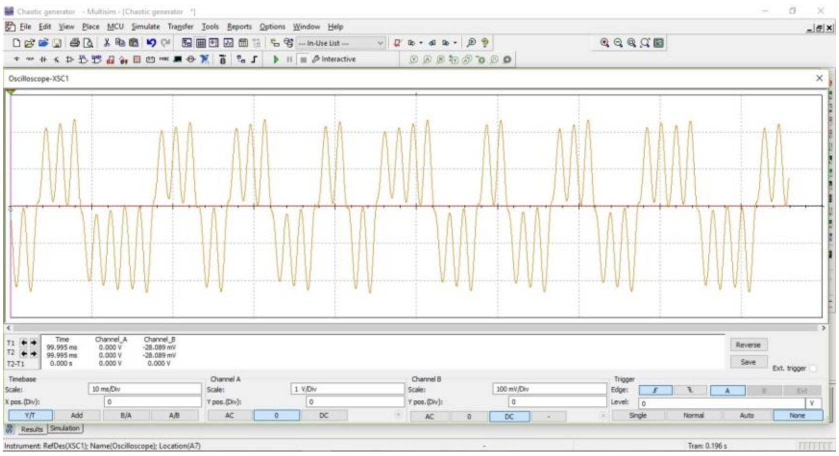

Figure 4. Time dependence of the coordinate Y

Depending on the setting of the bifurcation parameter R, the primary observed frequency will be in the range of 350±550 Hz. At some settings of *R*, there will also be strong low-frequency components (in the range of 2 Hz) as the trajectory slowly jumps between lobes.

As *R* is increased, the real eigenvalue migrates toward the origin. For the complex conjugate eigenvalues, ω decreases in magnitude as R is increased, while σ crosses into the right half-plane around R = 15 kΩ, increases to a maximum of 106 around R = 20 kΩ, and then decreases and returns to the left half-plane around R = 70 kΩ. Referring to the bifurcation diagram Figure 5, the onset and the demise of chaos correspond to $\sigma > 0$.

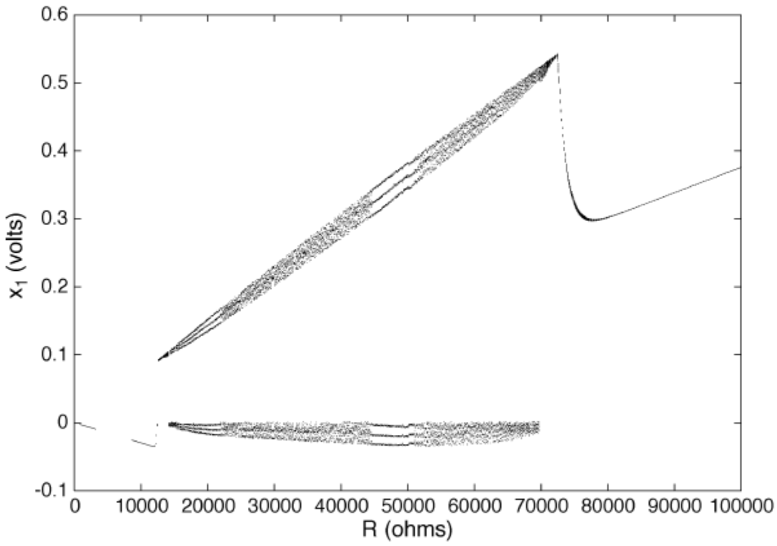

Figure 5. Theoretical bifurcation diagram

For the chaotic attractor shown in Figure 2, the Lyapunov exponent spectrum was L_1=0.1880, L_2=0, L_3=-7,045. There is one positive Lyapunov exponent, so the system is chaotic [15-21]. In addition, the Kaplan–Yorke dimension of the system is found as:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.027 .$$

3 Control of the CCC generator

There are many different approaches or techniques have been proposed to achieve chaos control, such as OGY approach, linear feedback control, inverse optimal control, among many others. It is well known that the theoretical basis of most known methods is stabilizing the unstable periodic orbits via parameter perturbation [1].

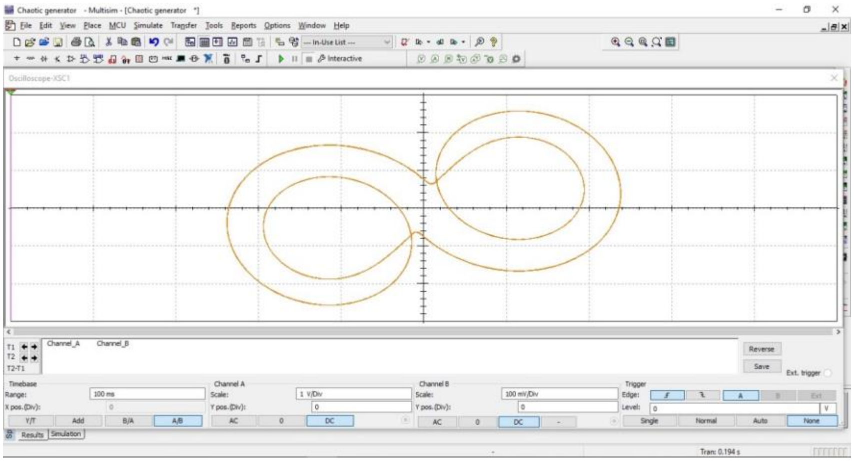

Fig. 6. 2-period control attractor

For control only one capacitor C2 was exchanged. If C2 = 20 nF we can got 2-period control attractor. Figure 6 shows the result of 2-period control attractor. In Figure 7 and Figure 8 shows time dependences of the coordinates X and Y respectively.

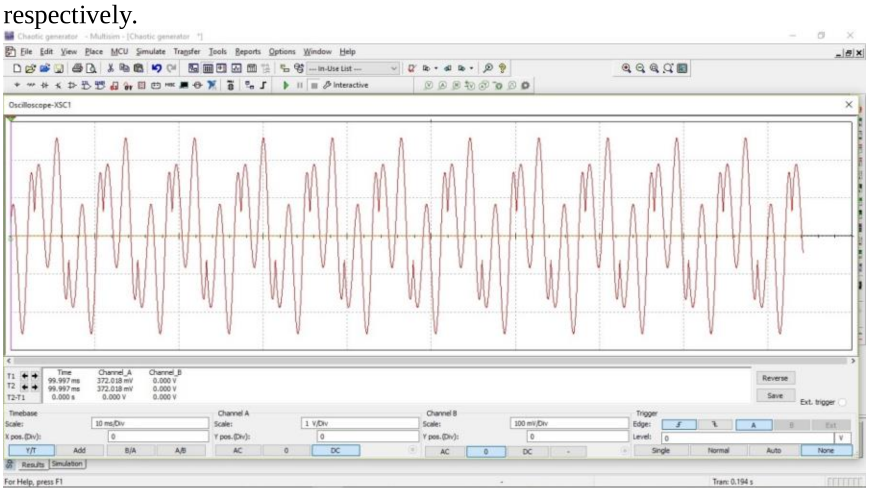

Figure 7. Time dependence of the coordinate X

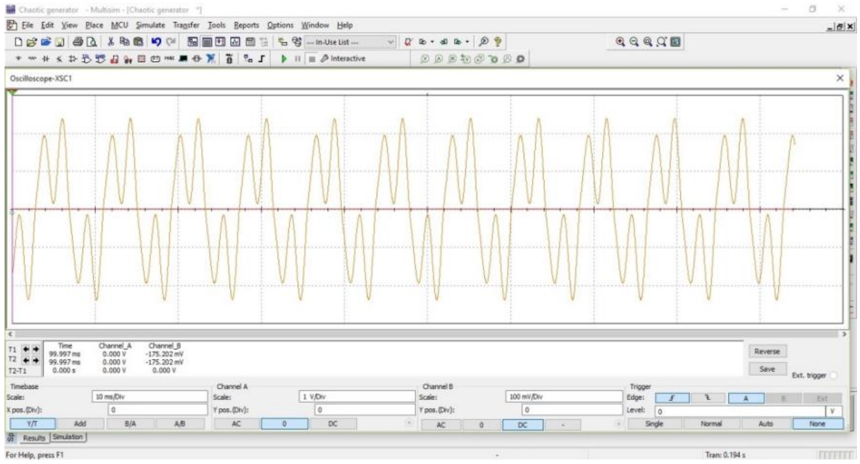

Figure 8. Time dependence of the coordinate Y

If C2 = 14 nF we can get 3-period control attractor. Figure 9 shows the result of 3-period control attractor.

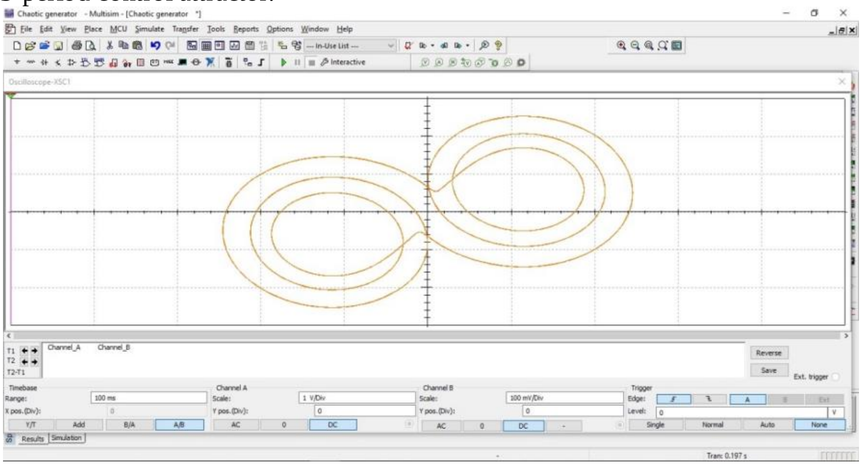

Figure 9. 3-period control attractor

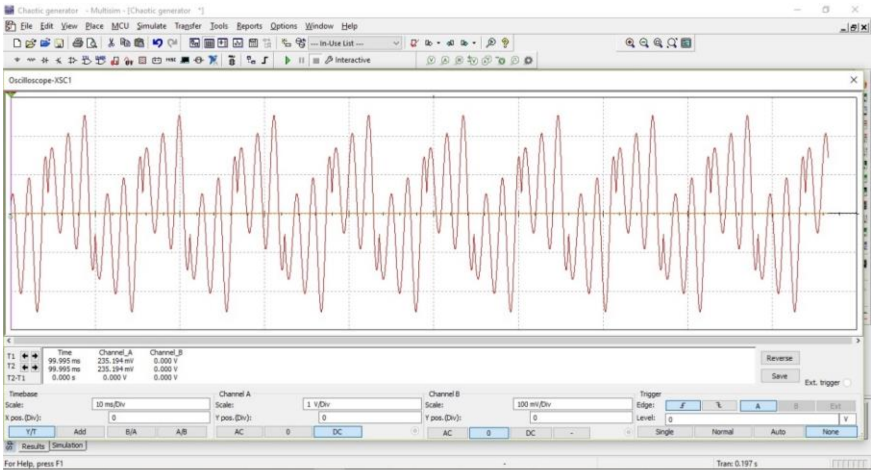

Figure 10. Time dependence of the coordinate X

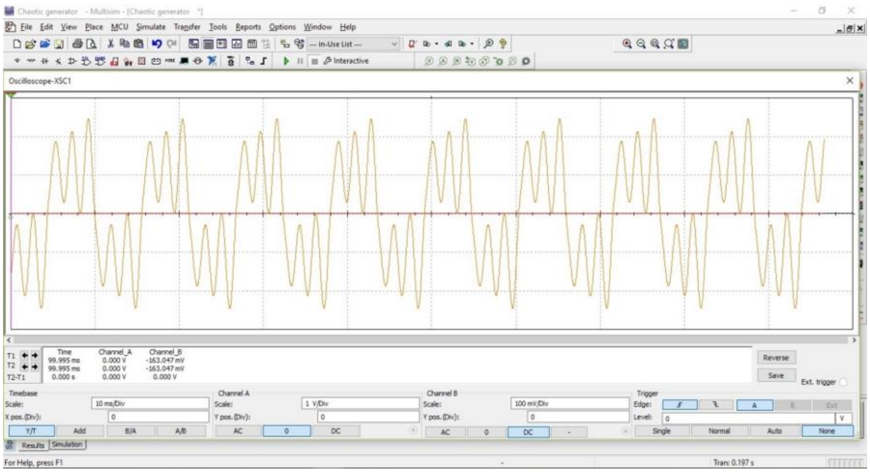

Figure 11. Time dependence of the coordinate Y

In Figure 10 and Figure 11 shows time dependences of the coordinates X and Y respectively.

Conclusions

The circuit equations are the simplest because there is no locally active resistor (R) in the circuit, where the capacitor (C) is the control parameter. The attractive features of this circuit are the presence of period-doubling route to chaos, period-doubling window through strong chaos followed by boundary condition etc. It is of further interest to study these aspects also in this system as well as the intermittency route to chaos and synchronization of coupled chaotic circuits of the present system for improved high security communication systems.

For the first time was conducted control CCC circuit. For demonstration was applied modern software environment MultiSim. The values of capacitor C2 can be used as keys for masking and decryption of information carrier in modern systems transmitting and receiving information.

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