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Cardiorespiratory System as Nonideal System with Limited Excitation

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Abstract: A new modified cardiorespiratory model based on the famous DeBoer beat-to-beat model and Zaslavsky map (which describes dynamics of the respiratory system as a generator of central type) was studied in details. The respiratory tract was firstly modeled by a self-oscillating system under the impulsive influence of heartbeat and cardiovascular system was represented as an oscillating system with a limited excitation. Chaotic modes were revealed, which were produced by the interaction between the subsystems. It was proved that the irregularity of the behavior of phase trajectories depends on the intensity of the effect produced by the heart rate on breathing, which is characteristic of the dynamics of the cardiovascular system of a healthy person.
KeyWords: Heart rate, Cardiovascular system, Respiratory system, Feedback, Chaos.

1 Introduction

The human cardiovascular system directly and indirectly interacts with different systems of entire organism. Realized self-oscillations in a cardiovascular system are under an activity of practically all organs (see [2, 3, 5, 6, 15, 17, 18]). There are numerous interactions of heart rhythms between itself and with an internal and external environment. Cardiac and respiratory rhythms form up during embryo stage, and even the brief break of these rhythms after a birth results in death.

Existence of breathing and heart rhythm synchronization effect, found experimentally in the cardiovascular system both for healthy people and with pathologies, is well-proven in work Toledo [17] in 2002. It is well known, the dynamic process of mutual synchronization can be realized only in a case of presence of a subsystem mechanical interaction. Therefore, the indicated effect display testifies the presence of both direct and feedback interactions between the cardiovascular and respiratory systems.

A heart system and organism of man in general have one of major descriptions of activity, such as a blood pressure dynamics. His time-history, along with electrocardiogram (ECG), is an important information generator for research and diagnostics of laws and pathologies of the cardiovascular system. The task of mathematical model construction, describing the dynamics of arterial blood pressure, is far from completion. Complications of such design are related to the

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necessity of taking into account of influence on the cardiac rhythms not only the cardiovascular system but also other organs and systems of organism, in particular a respiratory system. According to studies in healthy people, the heart rate is on average about 60 beats / min and can fluctuate within 20 beats / min for every few beats. During the day, the heart rate can vary from 40 to 180 beats / min. The novelty of this paper is that we use the DeBoer model of a cardiovascular system with heart beats around 60 beats / min and compare results for the model with 75 beats / min.

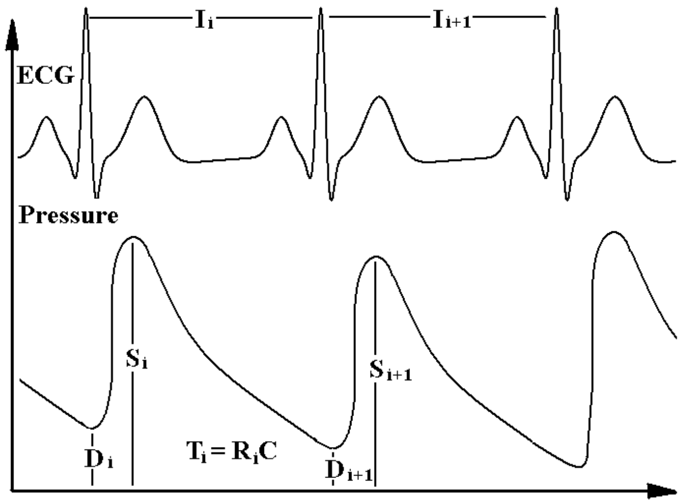

Fig. 1. Characteristics of the heartbeat in DeBoer model.

2 The mathematical model of direct and feedback interactions

The DeBoer model of a cardiovascular system is under direct action of a respiratory systems (what corresponds to experimental data) [3]. This model was substantially developed in future. The sinus node responsiveness (and other detailed factors) is taking into account in the work of Seidel and Herzel [15] (the so-called SH-model). In this model chaotic dynamics was found in dynamics of a cardiosystem.

The models of both DeBoer and SH only considered direct respiratory influence on heartbeats. The SH-model got further development [6], where an effect of heartbeat and the resultant changes in the baroreceptor afferent activitiy to the SH-model are added and the cardiorespiratory sinchronization found due to this modification. Interaction of blood pressure and amplitudes of breathing

oscillations revealed in accordance with principles of optimum control in the DeBoer model is investigated in the Grinchenko-Rudnitsky model [2]. This model allowed, in particular, to explain appearance of a peak on the Meyer frequency in the spectrums of pressure oscillations and synchronization of cardiac and respirator rhythms.
However, this model does not consider the reverse mechanical influence effect of the heartbeat changes on a breathing phase (frequency). In the present study, we add to the DeBoer model a self-oscillating system (which describes dynamics of the respiratory system as a generator of central type [5], shown in Figure 2) which is under impulsive influence of heartbeat.

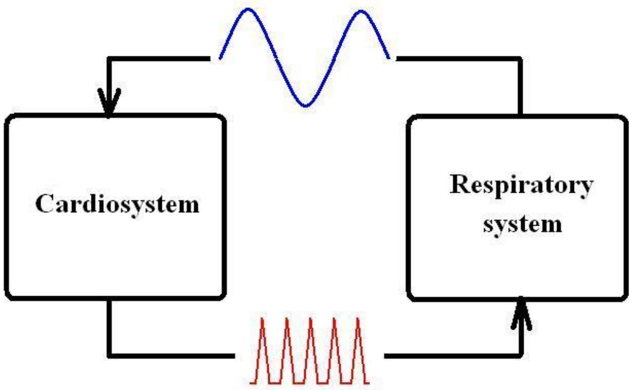

Fig. 2. Interaction of the cardiovascular and respiratory system

The DeBoer model describes the followings main characteristics of the heartbeat system: systolic pressure S , diastolic pressure D, R-R interval I and arterial time constant T (in a state of rest for a healthy man S=120 mmHg, D=80 mmHg, I=800 ms, T=1500 ms). This mathematical model is a system of five discrete nonlinear maps. This model contains only a direct mechanical influence of the respirator system on the cardiosystem and can be written in the form:

$$D_i' = c_1 S_{i-1}' \exp\left(-\frac{I_{i-1}'}{T_{i-1}'}\right),$$

$$S_i' = D_i' + \gamma \frac{T_0}{S_0} I_{i-1}' + \frac{A}{S_0} \sin\left(2\pi f T_0 t_i\right) + \frac{c_2}{S_0},$$

$$I_i' = G_v \frac{S_0}{T_0} \hat{S}'_{i-\tau_v} + G_\beta \frac{S_0}{T_0} F(\hat{S}', \tau_\beta) + \frac{c_3}{T_0},$$

$$T_i' = 1 + G_\alpha \frac{S_0}{T_0} - G_\alpha \frac{S_0}{T_0} F(\hat{S}', \tau_\alpha),$$

$$\hat{S}_i' = 1 + \frac{18}{S_0} \arctan \frac{S_0 (S_i' - 1)}{18},$$

where $i \geq 1$, $D' = D / S_0$, $S' = S / S_0$, $\hat{S}' = \hat{S} / S_0$, $I' = I / T_0$, $T' = T / T_0$, $F(\hat{S}, \tau) = 1/9(\hat{S}_{i-\tau-2} + 2\hat{S}_{i-\tau-1} + 3\hat{S}_{i-\tau} + 2\hat{S}_{i-\tau+1} + \hat{S}_{i-\tau+2})$, $t_i = \sum_{k=0}^{i-1} I_k'$ is a real time, A=3 mmHg is a breathing amplitude, f=0.25 Hz is a breathing frequency, $c_1 = D_0 / S_0 \exp(I_0 / T_0)$, $c_2 = S_0 - D_0 - \gamma I_0$, $c_3 = I_0 - S_0 (G_v + G_\beta)$, $\gamma = 0.016$ mmHg, $G_\alpha = 18$ ms/mmHg, $G_\beta = 9$ ms/mmHg, $G_v = 9$ ms/mmHg, $\tau_\alpha = \tau_\beta = 4$, $\tau_v = 0$, is equal to 0 if frequency of heartbeat is less then 75 beat/min, and τ_v is equal to 1, if frequency is more then 75 beat/min.

Generation of body rhythms, according to the theory of Glass [5] is carried out by the generators of the central type. Breathing is related to the movement of the chest and its dynamics can be modeled as the dynamics of the central generator. We suppose that a healthy man at rest breathes periodically with a permanent frequency and an amplitude of motions of thorax. In that case a breathing process can be described as the self-oscillating system, which has a steady limit circle. Thus for the mathematical modeling of a such system equations of the Zaslavskiy map could be used. Famous Zaslavsky map is the system of equations [14, 19] which describes the dynamics of an amplitude r_n and a phase φ_n of the system (in which periodic self-oscillations with a frequency ω are realized) which is under T-periodic impulsive action of constant intensity η .

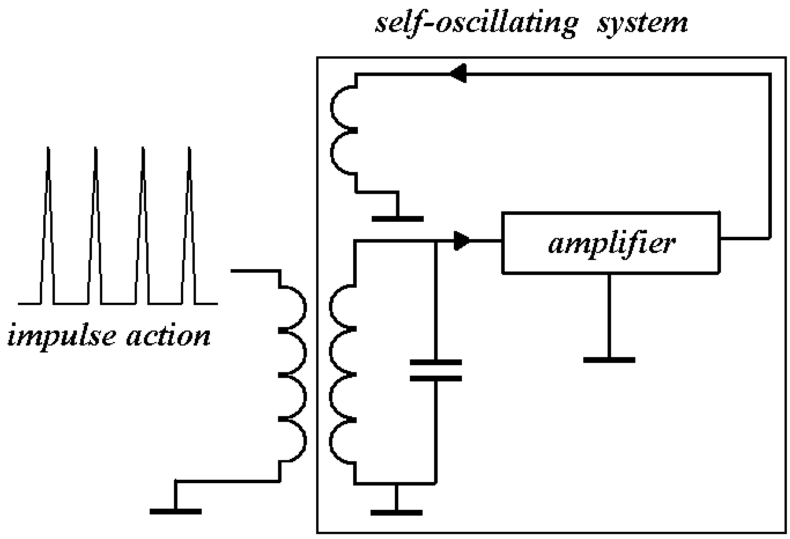

Fig.3 Schema Zaslavskiy generator as self-oscillating system with amplifier under impulse action

The system of Zaslavskiy map has the following form:

$$r_{n+1} = \left(r_n + \eta \sin \varphi_n\right) \exp\{-\kappa T\},$$
$$\varphi_{n+1} = \varphi_n + \omega T + \nu \left(r_n + \eta \sin \varphi_n\right) \frac{1-\exp\{-\kappa T\}}{\kappa},$$

where κ, ν are constant parameters.

In our approach these equations are used to describe changes of an amplitude and phase of a respiratory system effect for every R-R interval with an intensity proportional to systolic pressure: $\tilde{\eta} = -\eta(S_n - S_0)$:

$$r_{n+1} = \left(r_n - \eta(S_n - S_0) \sin \varphi_n\right) \exp\{-\kappa I_n\},$$
$$\varphi_{n+1} = \varphi_n + 2\pi f I_n + \nu \left(r_n - \eta(S_n - S_0) \sin \varphi_n\right) \frac{1-\exp\{-\kappa I_n\}}{\kappa},$$

where I is R-R interval, $\eta > 0$, κ, ν are constant parameters of interaction.

Thus, we study the dynamics of the modified model of cardiorespiratory system, which consists of the DeBoer model with direct respiratory influence $(A+r_i)\sin\varphi_i$, and with reverse influence modeled by the Zaslavskiy map system (see Figure 3). In other words, we consider cardio subsystem as oscillating system with a limited excitation from the respiratory subsystem [1, 4, 7-13, 16]. This is the novelty of our approach.

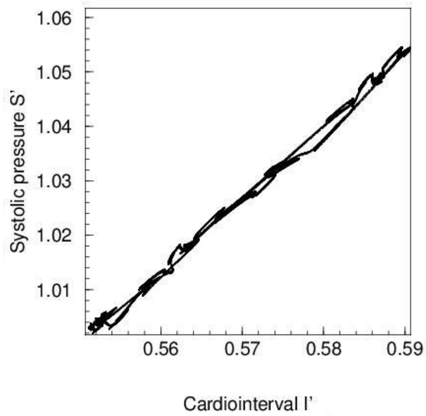

Fig. 4 Chaotic attractor projection in the phase space for $\eta = 0.25$

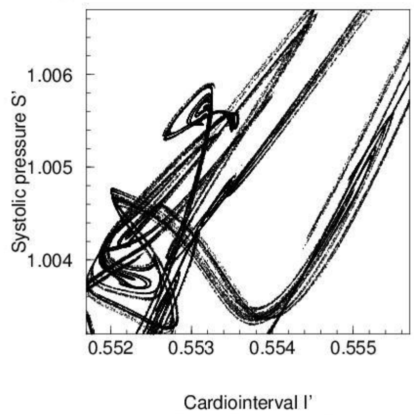

Fig. 5. An enlarged fragment of the projection of the chaotic attractor phase portrait for $\eta = 0.25$

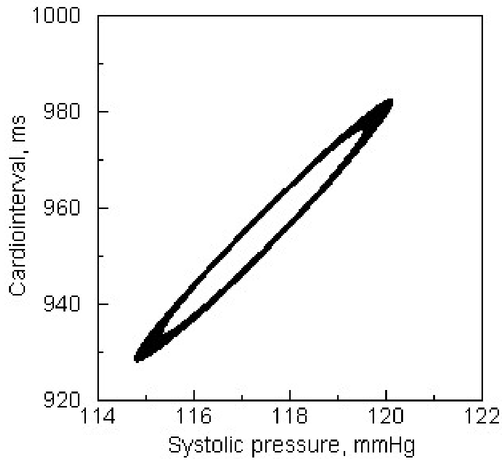

Fig. 6. Projection of the quasiperiodic regime of the model at $\eta = 0.01$ and the heart rate equals 60 beats per minute

3. Numerical simulations results

In accordance with physiology of healthy man, the followings values of variables and constants are used in our numerical simulations:

$I'[0] = 0.53$, when the pulse value is 75 beats per minute,

$I'[0] = 0.67$, when the pulse value is 60 beats per minute,

$$S'[-j] = 1.08, \quad j = 0, ..., 6,$$

$$r'[0] = 0, \ \varphi'[0] = 0, \ \kappa = 0.001 \text{ 1/ms}, \ \nu = 0.001 \text{ 1/msmmHg}.$$

In order to study steady-state regimes first of all the largest Lyapunov exponent [1, 7-13] was found. There is the region where Lyapunov exponent positive and that means transition to chaos occurs. We emphasize that η describes intensity of heart influence on a respiratory system.

When the pulse value is 75 beats per minute transition to chaos occurs at the condition that $\eta > 0.245$. Phase portrait projection on the phase space of the simulated systolic pressure and R-R interval data is presented in Figure 4 for the pulse value is 75 beats per minute. And an enlarged fragment of the structure of this attractors is shown in Figure 5 where complicated structure is obvious. If we now reduce the pulse value to 60 beats per minute, then the behavior of the system is different. Now transition to chaos is at different values, namely at $\eta = 0.17$. For the smaller values of η the hard transition to chaos disappears, instead the system is characterized by a number of quasiperiodic attractors (Fig. 6). The phase portrait in the Figure 6 represents a solid curve and corresponds to quasiperiodic regime at $\eta = 0.01$. Phase portrait projection of the chaotic attractor at $\eta = 0.17$ and an enlarged fragment of the structure of this attractors are presented in Figure 7.

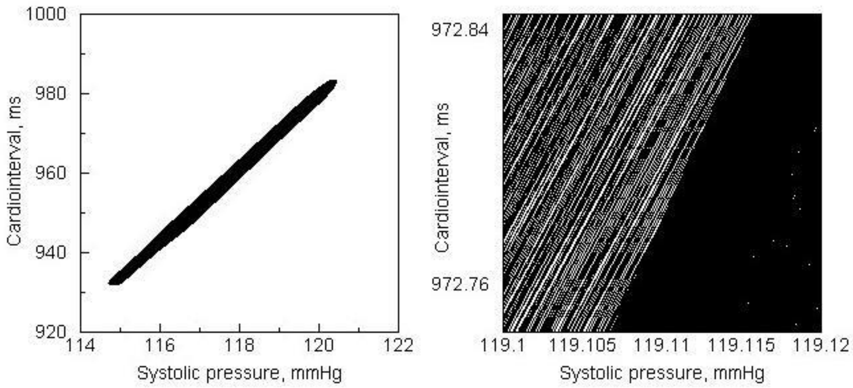

Fig.7 Phase portrait projection of the chaotic attractor at $\eta = 0.17$ and an enlarged fragment of the structure of this attractors for the case of a pulse of 60 beats per minute.

4. Conclusions

On the basis of the DeBoer model an interaction of the heartbeat and the respiratory system as dissipative Zaslavskiy map is studied and modeled as the system with limited excitation. This model takes into account both direct and reverse influence of subsystems – cardiovascular and respiratory.

The methods of modern theory of the dynamical systems are used to study laws of the steady-state regimes of the modified model. It is established that the irregularity of the behavior of the phase trajectories depends on the intensity of the action of the heart rhythm on breathing, which is characteristic of the dynamics of the cardiorespiratory system of a healthy person. The sensitivity of the model to the pulse rate (cardio-interval) was also revealed, namely simplification of the attractor structure of the whole system under reducing the pulse rate from 75 to 60 beats per minute, which corresponds to the generally accepted norms of activity of the human body.

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