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Hidden Bifurcation kind to Multiscroll Chaotic Attractors via Saturated Function Series

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Abstract. In this paper, design Hidden Bifurcation kind to Multiscroll Chaotic Attractors via Saturated Function Series are reconnoitred. The idea was taken from the work Zaamoune, et al(2019) and the method introduced by Menacer, et al. (2016) for Chua multiscroll attractors. These idea (hidden bifurcation kind) depends on how you appear the scrolls odd or even were the number of scrolls is even. We have studied many examples to prove this idea, mentioned that the number n of scrolls satisfies $n = p + q + 2$ in LÜ , Chen et al (2004).
Keywords: Hidden bifurcation, multiscroll chaotic attractor, saturated function series..

1 Introduction

It is well known that the method hidden attractor generating one-directional(1-D) -scroll has been studied in the last few years [1]-[3]-[4]. Since the method was presented by Leonov, et al [4] in the Chua attractor, they have proposed efficient technique for the numerical localization of the hidden attractors in one-directional dynamical systems. Hidden attractor has many applications in a real word like mechanics, electronics, chemistry, biology but the most important in electronic circuits (hysteresis circuit, and saturated circuit). In 2016, the auteur's Menacer, et al changed the type of discrete parameters by presented a generating multi-spirals, and this new method they called a "hidden bifurcation", the cause of this name it's has a change in the number of spirals. In this paper, we study design hidden bifurcation kind method in the attractor for generating one-directional (1-D) -scroll by saturated function series. In 2004 Chen, et al design and analysis of multi scroll chaotic attractors from saturated function series [8], but we present a new idea in hidden bifurcation, where we know that the role of method hidden bifurcation it's a parametres control in the number of spirals, her in this work p_1 and q_1 in function 12 it's a bossed. In this article, we change the values of system Chen parameters 11 a, b ,c and d_1 we found a new attractor different about Chen attractor in [8], we change every

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time the parameter ϵ in 0.55 to 1 and the parameters p_1 and q_1 and we noted that it's not only the parameter ϵcontrol in the appearance of numbers scrolls, the parameters p_1 and q_1 also control in appearance of numbers scrolls it's a new idea. This paper is disposed of as follows : In Section 2, the analytical-numerical method for hidden attractor proposed by Leonov in [6],[7]. the model of $1-D$ scroll chaotic attractors generated by saturated function series proposed .In Section 3, the model of $1-D$ scroll chaotic attractors generated by saturated function series proposed. In section 4 , the localization technique presented in [2] for hidden bifurcation in $1 - D$ scroll chaotic attractors, we introduced the results for a new idea. Finally, in Sec. 6, a terse conclusion is pictured.

2 Analytical-numerical procedure for attractors localization

Leonov et al. [6],[9], [10] found a procedure to discover numerically hidden attractor for Chua attractor. The technique developed in [2], discovering hidden bifurcations in the multispiral Chua attractor. *To improv, this numerical method, consider a system with one-directional (1-D) -scroll*

$$\frac{dx}{dt} = Px + \beta\Psi(\kappa^T x), \quad x \in \mathbb{R}^3. \tag{1}$$

were P is a constant $(n \times n)$ matrix,β, κ are constant $n-$dimensional vectors, T is a transposition operation, $\Psi(\varsigma)$ is a continuous piecewise-differentiable vector-function, and $\Psi(0) = 0$. Consider a coefficient of Harmonic linearization k at like the matrix P_0 as :

$$P_0 = P + k\beta\kappa^T \tag{2}$$

wich $\pm i\omega_0$ $(\omega_0 > 0)$ eigenvalues the matrix P_0 and the rest have negative real parts. Suppose that such k occurs. So, rewrite system 1 as.

$$\frac{dx}{dt} = P_0 x + \beta\varphi(\kappa^T x), \tag{3}$$

were $\varphi(\varsigma) = \Psi(\varsigma) - k\varsigma$. We display a fixed sequence of functions $\varphi^0(\varsigma), \varphi^1(\varsigma), \cdots, \varphi^n(\varsigma)$, that the function $\varphi^0(\varsigma)$ is small, and $\varphi^m(\sigma) = \varphi(\varsigma)$. In this state the smallness of function $\varphi^0(\varsigma)$, permit one to practise the procedure of harmonic linearization for the system

$$\frac{dx}{dt} = P_0 x + \beta\varphi^0(\kappa^T x) \tag{4}$$

and conclude a stable nontrival periodic solution $x^0(t)$. So, the localization of the attractor of a system(19), design numerically the transformation of this periodic solution. So, we obtain the primary condition $x^0(0)$ of the periodic solution, system (4) can be changed by S $(X = SY)$ to the form :

$$\begin{cases} \dot{y_1} = -\omega_0 y_2 + v_1\varphi^0(y_1 + c_3^t Y_3) \\ \dot{y_2} = \omega_0 y_1 + v_2\varphi^0(y_1 + c_3^t Y_3) \\ \dot{Y_3} = A_3 Y_3 + V_3\varphi^0(y_1 + c_3^t Y_3) \end{cases} \tag{5}$$

So V_3 and c_3 is an $(n-2)$ −dimensional vector, y_1, y_2 are scalar values; Y_3 is an $(n-2)$ −dimensional vector, v_1 and bv_2 are real numbers; A_3 is an $(n-2)\times(n-2)$ matrix, where all of its eigenvalues have negative real parts. Supposed that for the matrix A_3 there exists a $d_3 > 0$ such that

$$Y_3^t(A_3 + A_3^t)Y_3 \leq -2d_3 \left|Y_3\right|^2, \ \forall Y_3 \in \mathbb{R}^{n-2}. \tag{6}$$

In one-directional(1-D) -scroll, case, present the describing function Φ of a real variable ς as follows:

$$\Phi(\varsigma) = \int_0^{2\pi/\omega_0} cos(\omega_0 t)\varsigma)\cos((\omega_0 t)\, dt \tag{7}$$

Theorem 1. *If a positive ς_0 satisfies that*

$$\Phi(\varsigma_0) = 0, \ b_1 \frac{d\Phi(\varsigma)}{d\varsigma}\bigg|_{\varsigma=\varsigma_0} < 0 \tag{8}$$

So, for the initial condition of the periodic solution $X^0(0) = S(y_1(0),\ y_2(0), Y_3(0))^T$ at the first step of algorithm, one has

$$y_1(0) = \varsigma_0 + O(\epsilon), \ y_2(0) = 0, \ Y_3(0) = O_{n-2}(\epsilon) \tag{9}$$

In Application, to find k and ω_0, can uses the transfer function of system (4),

$$W(\rho) = \kappa^T(M - \rho I)^{-1}\beta \tag{10}$$

where ρ is a complex variable. The number ω_0 is calculated from the equation $ImW(i\omega_0) = 0$ and k is then determined using the formula $k_0 = -ReW(i\omega_0)$.

3 $1-D$ Scroll Chaotic Attractor via a saturated function series

Here, to generate $1-D$ n scroll chaotic attractor we presented a system from saturated function series follows :

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -ax - by - cz + d_1 f(x; k_1; h_1; p_1; q_1) \end{cases} \tag{11}$$

where

$$f(x; k_1; h_1; p_1; q_1) = \begin{cases} y_1 & if\ x > q_1 h_1 + 1 \\ y_2 & if\ \left|x - ih_1 \leq 1, -p_1 \leq_1 i \leq q_1\right| \\ y_3 & if\ ih_1 + 1 < x < (i+1)\,h_1 - 1 \\ \text{and} & -p_1 < i < q_1 - 1 \\ y_4 & if\ x < -q_1 h_1 - 1 \end{cases} \tag{12}$$

$y_1 = (2q_1 + 1)\,k_1$, $y_2 = k_1(x - ih_1) + 2ik_1$, $y_3 = (2i+1)\,k_1$, and $y_4 = -(2q_1+1)\,k_1$here, a, b, c, d_1 are real numbers and the parameters p_1, q_1, h and k are integers. The formula to calculated the number n of scrolls, it's explained in Chen, et al [8] and in Zaamoune, et al[11] For $k = 11$,$h = 22$,$p_1 = 3$, $q_1 = 3$, $a = d_1 = 0.8$, $c = 0.72$ and $b = 0.6$ a 8-scroll attractor is generated as the verged attractor of the system (11-12), see Fig 1

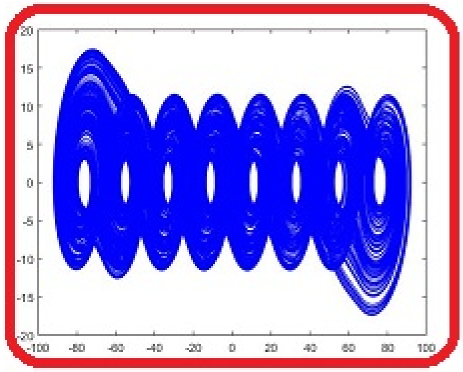

Fig. 1. One-directional 8-scroll chaotic attractors.

4 Generalized hidden bifurcation with scalar nonlinearity.

Now, we apply the above procedure; rewrite a system 11 in the form

$$\frac{dx}{dt} = Px + \beta\Psi(\kappa^t x), \qquad x \in \mathbb{R}^3 \tag{13}$$

Here

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

$$\kappa = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \beta \begin{pmatrix} 0 \\ 0 \\ d_1 \end{pmatrix} \quad and \quad \psi(\varsigma) = f(\varsigma).$$

Define the coefficient k and a small parameter ϵ, system (13) can be rewrite in the forme :

$$\frac{dx}{dt} = P_0 x + \beta\epsilon\varphi(\kappa^T x), \tag{14}$$

where

$$P_0 = P + k\beta\kappa^T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ k_0 d_1 - a & -b & -c \end{pmatrix},$$

$$\lambda_{1,2}^{P_0} = \pm i\omega_0, \quad \lambda_3^{P_0} = -d$$

The transfer function $W_P(\lambda)$ of system (14) can be given by

$$W_{P_0}(\lambda) = \kappa^T(P - \lambda I)^{-1}\beta \tag{15}$$

where λ is a complex variable. By the transformation $X = SY$, system (14) is changed to the form

$$\frac{dY}{dt} = HY + v\varepsilon\varphi(c^T Y) \tag{16}$$

where

$$H = \begin{pmatrix} 0 & -\omega_0 & 0 \\ \omega_0 & 0 & 0 \\ 0 & 0 & -d \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ Y_3 \end{pmatrix}, \quad v = \begin{pmatrix} v_1 \\ v_2 \\ 1 \end{pmatrix}$$

and $c = \begin{pmatrix} 1 \\ 0 \\ -h \end{pmatrix}$. The transfer function of system (16) can be written as

$$\begin{aligned} W_H(\lambda) = & \quad c^T(H - I\lambda)^{-1}v \\ & = \tfrac{h}{d+\lambda} - \lambda\tfrac{v_1}{\lambda^2+\omega_0^2} + \omega_0\tfrac{v_2}{\lambda^2+\omega_0^2} \end{aligned}$$

So, we could obtain the implies k, d, h, v_1, v_2 by using the equality of transfer functions of systems (14) and (16):

$$W_H(\lambda) = \kappa^T(M_0 - \lambda I)^{-1}\beta \tag{17}$$

This implies the following relations:

$$\begin{aligned} & k_0 = \tfrac{a-\omega_0^2 d}{d_1} \\ & d = c \\ & h = \tfrac{-d_1}{\omega_0^2+d^2} \\ & v_1 = \tfrac{-d_1}{\omega_0^2+d^2} \\ & v_2 = \tfrac{-cd_1}{\omega_0(\omega_0^2+d^2)} \end{aligned} \tag{18}$$

We utilized the transformation $X = SY$the following relationships can be gotten :

$$H = S^{-1}P_0S,\ b = S^{-1}\beta,\ c^T = \kappa^t S \tag{19}$$

So, by 19 we found this matrix :

$$S = \begin{pmatrix} S_{11} = 1 & S_{12} = 0 & S_{13} = -h \\ S_{21} = 0 & S_{22} = -\omega_0 & S_{23} = dh \\ S_{31} = -\omega_0^3 & S_{32} = 0 & S_{33} = d^2h \end{pmatrix}$$

So, thr first step in above procedure is determine the initial data, as

$$X(0) = SY(0) = S\begin{pmatrix} \varsigma_0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \varsigma_0 S_{11} \\ \varsigma_0 S_{21} \\ \varsigma_0 S_{31} \end{pmatrix} \tag{20}$$

For system the Chen, the initial condition is :

$$X^0(0) = \left(x^0(0) = \varsigma_0,\ y^0(0) = 0,\ z^0(0) = -\varsigma_0\omega_0^3 \right) \tag{21}$$

5 Numerical Results of Hidden Bifurcation kind

In this work, we introduced a new idea in hidden bifurcation behavior in the attractor by we changed the values of parameters of the system generated via saturated function serie. So, we presented the system (11-12) with parameter values

$$a = d_1 = 0.8, c = 0.72, b = 0.6p_1 = q_1 = 3, k = 11andh = 22$$

By folowing the above method we are started first calculation the frequency ω_0 and a coefficient of harmonic linearization k as well :

$$\omega_0 = 0.7745 \quad and \qquad k = 0.46$$

Then, we presented fours cases numbers of scrolls $4, 6, 8, 10,$ by numerous sequentially ε from the value $\varepsilon = 0.55$ to $\varepsilon = 1.$ while in the case of $p = 8$ and $q = 0$,the initial conditions are : $(x_0(0) = -3.3256,\ y_0(0) = 0,\ z_0(0) = 1.5450.)$

When, $p = q = 3$ the hidden bifurcation kind is even as $2, 4, 6, 8$ and for $p = 0, q = 6$ the hidden bifurcation kind is odd as $1, 2, 3, 4, 5, 6, 7, 8$ that means the number of scrolls appearance is one by one, see figures (4, 5).

The case 4 scrolls, $p = q = 1$ the hidden bifurcation kind is even as $2, 4$and for $p = 0, q = 2$ the hidden bifurcation kind is odd as $1, 2, 3, 4$, see figures (2, 3).

The case 6 scrolls, $p = q = 2$ the hidden bifurcation kind is even as $2, 4, 6$ and for $p = 0, q = 4$ the hidden bifurcation kind is odd as $1, 2, 3, 4, 5, 6$.

The case 10 scrolls, $p = q = 4$ the hidden bifurcation kind is even as $2, 4, 6, 8, 10$ and for $p = 0, q = 8$ the hidden bifurcation kind is odd as $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.

The role idea of the hidden bifurcation kind was 'even or odd' if the number of the scrolls was even. This idea based on the principle idea of [11]. View that when the parameters p or q equalizes zero the hidden bifurcation behavior it was odd, as for, $p = q$ the hidden bifurcation behavior was even and it's explicated in tables below (1) and figures (2, 3, 4, 5)

$n = p + q + 2$	The number of scrolls is even
p or q value Zero	The scrolls appearance is odd
$p = q$	The scrolls appearance is even

Table 1. The behavior of hidden bifurcation kind

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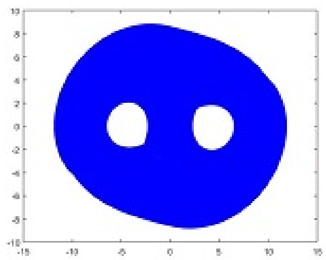
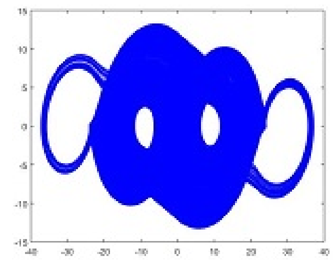
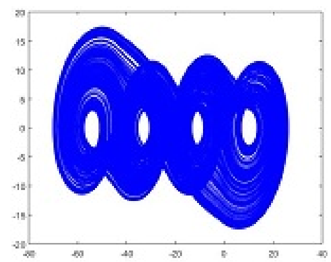


(a)

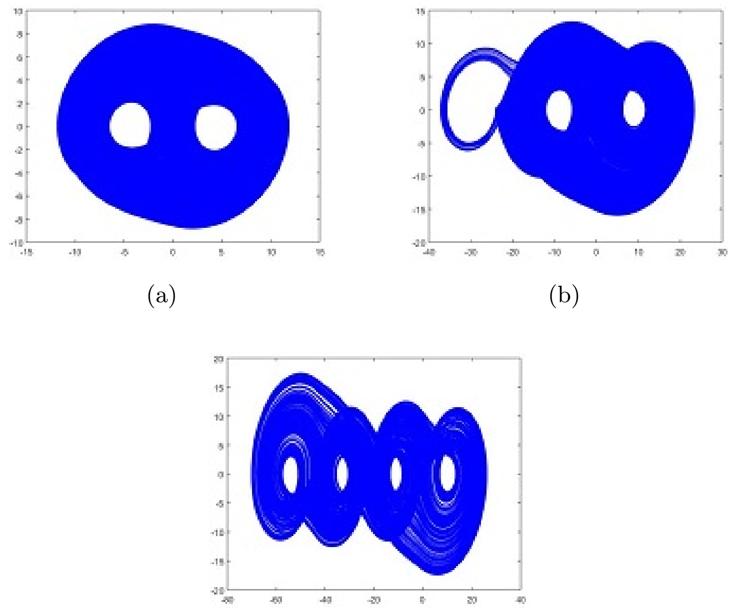


(b)



(c)

Fig. 2. The increasing number of spirals of system (14) according to increasing ε values, when $p = 1$ and $q = 1$, k=11 and h=22. (a) : The first scroll for ε=0.55, (b) : The second scroll for ε=0.92, (c) : The third scroll for ε=1.



(a) (b)

(c)

Fig. 3. The increasing number of spirals of system (14) according to increasing ε values, when $p = 2$ and $q = 0$, k=11 and h=22. (a) : The first scroll for ε=0.55, (b) : The second scroll for ε=0.92, (c) : The third scroll for ε=1.

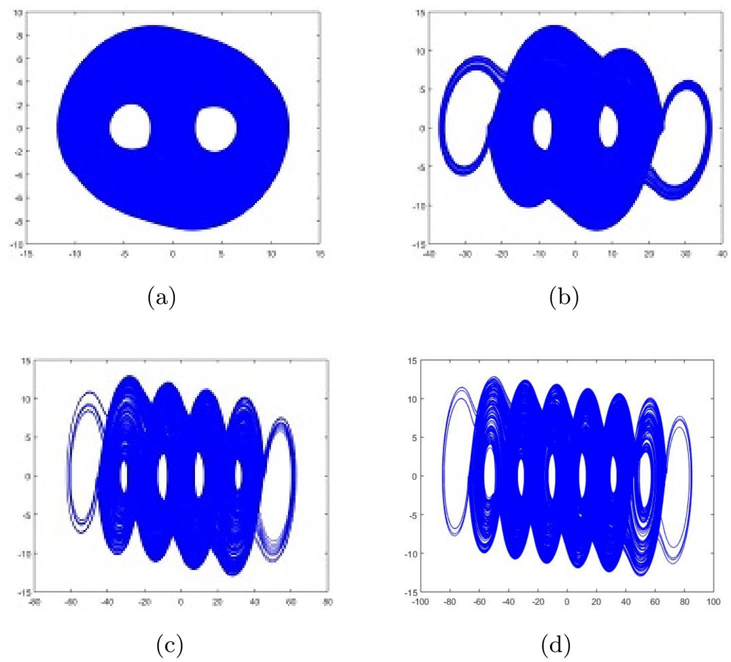


Fig. 4. The increasing number of spirals of system (14) according to increasing ε values, when $p = 3$ and $q = 3$, k=11 and h=22. (a) : The first scroll for ε=0.55, (b) : The second scroll for ε=0.92, (c) : The third scroll for ε=0.975, (d) : The last scroll for ε=1

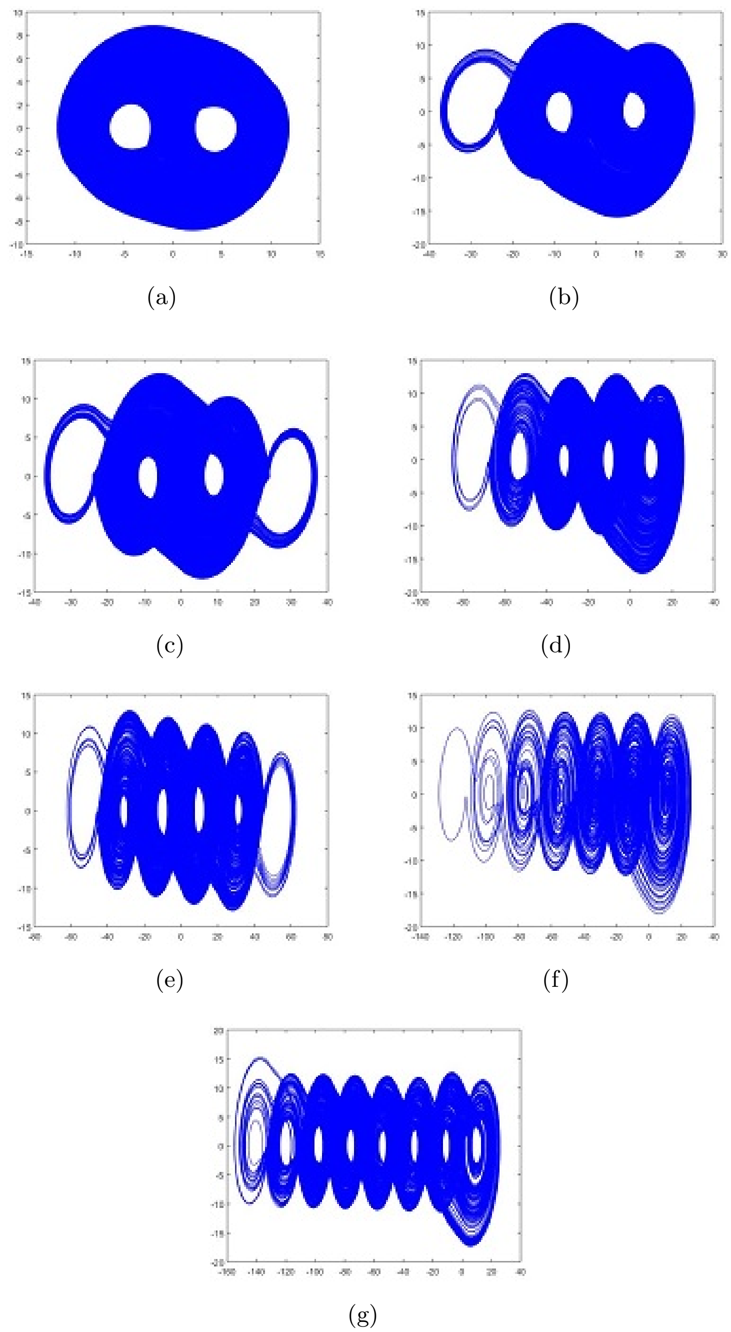


Fig. 5. The increasing number of spirals of system (14) according to increasing ε values, when $p = 6$ and $q = 0$, k=11 and h=22. (a) : The first scroll for ε=0.55, (b) : The second scroll for ε=0.92, (c) : The third scroll for ε=0.98, (d) : The fourth scroll for ε=0.985, (e) : The fifthly scroll for ε=0.99, (f) : The sixthly scroll for ε=0.992, (g) : The last scroll for ε=1